

# MATH 451/551

## Chapter 6. Joint Distribution

### 6.3 Joint Moment Generating Functions

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# Joint Moment Generating Functions



## Joint Moment Generating Functions

Let  $(X_1, X_2)$  be a random vector. The **joint moment generating function** of  $(X_1, X_2)$  is

$$M(t_1, t_2) = E(e^{t_1 X_1 + t_2 X_2})$$

provided that the expected value exists on  $-h_1 < t_1 < h_1$  and  $-h_2 < t_2 < h_2$  for some positive constants  $h_1$  and  $h_2$ .

- ▶  $E(X_1) = \left. \frac{\partial M(t_1, t_2)}{\partial t_1} \right|_{(t_1, t_2)=(0,0)}$ ,  $E(X_2) = \left. \frac{\partial M(t_1, t_2)}{\partial t_2} \right|_{(t_1, t_2)=(0,0)}$
- ▶  $E(X_1^2) = \left. \frac{\partial^2 M(t_1, t_2)}{\partial t_1^2} \right|_{(t_1, t_2)=(0,0)}$ ,  $E(X_2^2) = \left. \frac{\partial^2 M(t_1, t_2)}{\partial t_2^2} \right|_{(t_1, t_2)=(0,0)}$
- ▶  $E(X_1 X_2) = \left. \frac{\partial^2 M(t_1, t_2)}{\partial t_1 \partial t_2} \right|_{(t_1, t_2)=(0,0)}$
- ▶ Marginal moment generating functions:  
 $M_{X_1}(t_1) = E(e^{t_1 X_1}) = M(t_1, 0)$ ,  $M_{X_2}(t_2) = E(e^{t_2 X_2}) = M(0, t_2)$
- ▶ The random variables  $X_1$  and  $X_2$  are independent iff

$$M(t_1, t_2) = M_{X_1}(t_1)M_{X_2}(t_2)$$

# Example 1



## Example 1

Let  $X_1$  and  $X_2$  be continuous random variables whose distribution is described by the joint probability density function

$$f(x_1, x_2) = \lambda_1 e^{-\lambda_1 x_1} \lambda_2 e^{-\lambda_2 x_2}, \quad x_1 > 0, x_2 > 0$$

for real positive parameters  $\lambda_1$  and  $\lambda_2$ . Find the joint moment generating function for  $X_1$  and  $X_2$  and use it to find  $E(X_1 X_2)$ .

# Thank You



THANK YOU!