Department of Mathematics
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MATH 451/551

Chapter 6. Joint Distribution

6.3 Expected Values

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Example 1

Example 1

For the random variables X and Y with joint probability density function
$$f(x,y) = 2, \quad x > 0,\ y > 0,\ x + y < 1,$$
find $V(E(Y|X))$.

Theorem

Theorem 6.11

If X and Y are random variables, then

$$E(X) = E\{E(X|Y)\}$$

when the expectations exist.

Theorem

Theorem 6.12

If X and Y are random variables, then

$$V(X) = E\{V(X|Y)\} + V\{E(X|Y)\}$$

when the expectations exist.

Example 2

The binomial distribution was introduced asthe number of successes in n independent Bernoulli trials, each with a probability of success p. The population mean and variance of $X \sim Bin(n, p)$ are

$$E(X) = np \quad \text{and} \quad V(X) = np(1 - p).$$

But what if an application arises where n is fixed but p is unknown? Such an application might arise in an area of quality control known as **acceptance sampling**, where a fixed number of items are sampled from a large lot. To be more specific, assume that p is no longer a fixed probability, but is itself a random variable. Furthermore, assume that you have so little information about the probability of success on each Bernoulli trial that you can only state that the probability of success P is a random variable that is equally likely to fall between 0 and 1. That is to say, $P \sim U(0, 1)$. The problem here is to find $E(X)$ and $V(X)$ when $X \sim Bin(n, P)$, where $P \sim U(0, 1)$.

THANK YOU!