Department of Mathematics
College of William & Mary

MATH 451/551

Chapter 6. Joint Distribution

6.3 Covariance & Correlation

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Theorem

Theorem 6.5

If X and Y are random variables with finite population variances and covariance, then

$$V(X + Y) = V(X) + V(Y) + 2Cov(X, Y).$$

Example 1

A fair coin is tossed twice. Let X be the number of heads that appear and Y be the number of tails that appear. Find the population variance of $X + Y$.

Theorem

Theorem 6.6

If X and Y are independent random variables, then $Cov(X, Y) = 0$.

Example 2

Example 2

Show that the random variables X and Y that are uniformly distributed over the support $\mathcal{A} = \{(x, y)|1 < x^2 + y^2 < 4\}$ have a population covariance 0 and are dependent random variables.

Theorem

Theorem 6.7

If X and Y are independent random variables, then

$$V(X+Y) = V(X) + V(Y).$$

Correlation

Correlation

Let X and Y be random variables with finite population means μ_X and μ_Y, and finite population variances $\sigma_X^2 > 0$ and $\sigma_Y^2 > 0$, respectively. The **population correlation** between X and Y is

$$\rho = \frac{E\{(X - \mu_X)(Y - \mu_Y)\}}{\sigma_X \sigma_Y}.$$

Theorem

Theorem 6.8

If X and Y are independent random variables, then $\rho = 0$.

Theorem

Theorem 6.9

If X and Y are random variables with population correlation ρ, then $-1 \leq \rho \leq 1$.

Example 3

Let the discrete random variables X and Y have joint probability mass function $f(x, y)$ given by the entries in the table. Find the population correlation between X and Y.

	1	2	3	$f_X(x)$
1	0.2	0.1	0.3	0.6
2	0.1	0.1	0.2	0.4
$f_Y(y)$	0.3	0.2	0.5	1

Theorem

Theorem 6.10

Let X and Y be random variables with population correlation ρ. The population correlation ρ equals -1 iff the support of X and Y lies on a line with negative slope. The population correlation ρ equals 1 iff the support of X and Y lies on a line with positive slope.

- A population correlation of -1 between the random variables X and Y is often known as a **perfect negative correlation**.
- A population correlation of 1 between the random variable X and Y is often known as a **perfect positive correlation**.

Example 4

A fair coin is tossed twice. Let X be the number of heads that appear and Y be the number of tails that appear. Find the population correlation between X and Y.

THANK YOU!