Department of Mathematics
College of William & Mary

MATH 451/551

Chapter 6. Joint Distribution

6.3 Covariance

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Covariance

Covariance

Let X and Y be random variables with finite population means μ_X and μ_Y, respectively. The population covariance between X and Y is

$$Cov(X, Y) = E\{(X - \mu_X)(Y - \mu_Y)\}.$$

- Symmetric in its arguments: $Cov(X, Y) = Cov(Y, X)$
- Defining formula useful for conceptualizing covariance
  - If X and Y tend to be on opposite sides of their means together $\Rightarrow$ population covariance negative
  - If X and Y tend to be on the same sides of their means together $\Rightarrow$ population covariance positive

Example 1

Example 1

A fair coin is tossed twice. Let X be the number of heads that appear and Y be the number of tails that appear. Find the population covariance between X and Y.

Special Case

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Variance is a special case of covariance: $V(X) = Cov(X, X)$.

- **Bivariate Case:**

$$\Sigma = \begin{pmatrix} V(X) & Cov(X, Y) \\ Cov(Y, X) & V(Y) \end{pmatrix}$$

- **Trivariate Case:**

$$\Sigma = \begin{pmatrix} V(X) & Cov(X, Y) & Cov(X, Z) \\ Cov(Y, X) & V(Y) & Cov(Y, Z) \\ Cov(Z, X) & Cov(Z, Y) & V(Z) \end{pmatrix}$$

Theorem 6.4

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If X and Y are random variables with finite population means μ_X and μ_Y, respectively, then

$$Cov(X, Y) = E\{(X - \mu_X)(Y - \mu_Y)\} = E(XY) - \mu_X \mu_Y.$$

Example 2

A fair coin is tossed twice. Let X be the number of heads that appear and Y be the number of tails that appear. Find the population covariance between X and Y using the shortcut formula

Example 3

Example 3

Deal two cards from a well-shuffled deck. Let X be the number of aces dealt and Y be the number of face cards dealt. Using the shortcut formula, find the population covariance between X and Y.

THANK YOU!