Department of Mathematics
College of William & Mary

MATH 451/551

Chapter 6. Joint Distribution

6.3 Expected Values

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Expected Values

Expected Values

Let X and Y be random variables with joint probability mass function $f(x, y)$ if the random variables are discrete or joint probability density function $f(x, y)$ if the random variables are continuous. The expected value of $g(X, Y)$ is

$$E\{g(X,Y)\} = \begin{cases} \sum_x \sum_y g(x,y) f(x,y) & X, Y \text{ discrete} \\ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) f(x,y) dy dx & X, Y \text{ continuous} \end{cases}$$

- The expected value of a function of two random variables $g(X, Y)$ is simply the sum or integral of the product of the functions $g(x, y)$ and $f(x, y)$.

Example 1

Find $E(X+Y)$ for the discrete random variables X and Y with joint pmf

$$f(x,y) = \begin{cases} 0.2 & x=1, y=1 \\ 0.1 & x=1, y=2 \\ 0.3 & x=1, y=3 \\ 0.1 & x=2, y=1 \\ 0.1 & x=2, y=2 \\ 0.2 & x=2, y=3 \end{cases}.$$

Example 2

Find $E(X^2 Y)$ for X and Y with joint pdf

$$f(x, y) = \frac{1}{50} \quad x > 0, y > 0, x + y < 10.$$

Theorem

Theorem 6.2

If X and Y are random variables, then

$$E\{g(X) + h(Y)\} = E\{g(X)\} + E\{h(Y)\}$$

for any functions g and h.

Theorem

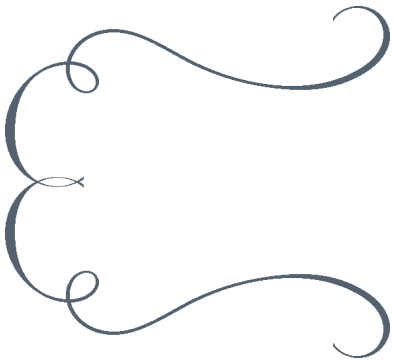
Theorem 6.3

If X and Y are independent random variables, then

$$E\{g(X)h(Y)\} = E\{g(X)\}E\{h(Y)\}$$

for any functions g and h.

Thank You



THANK YOU!