Department of Mathematics
College of William & Mary

# MATH 451/551

## Chapter 6. Joint Distribution

### 6.2 Independent Random Variables

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# Theorem


## Theorem 6.1

Let the random variables $X$ and $Y$ have joint distribution described by $f(x, y)$ defined on a product space. Then $X$ and $Y$ are independent iff $f(x, y)$ can be written as the product of a function of $x$ only and a function of $y$ only.

# Example 2 (Cont.)


## Example 2

Let $X_1$ and $X_2$ be random variables with joint pdf

$$f(x_1, x_2) = x_1 x_2, \quad 0 < x_1 < 1,\ 0 < x_2 < 2$$

Are $X_1$ and $X_2$ independent?


## Example 3

Are the random variables $X$ and $Y$ with joint probability density function

$$f(x,y) = \frac{1}{50}, \quad x > 0,\ y > 0,\ x + y < 10$$

independent?


## Example 4

Are the random variables $X$ and $Y$ with joint probability density function

$$f(x,y) = xy - 2x - y + 2, \quad 0 < x < 1,\ 0 < y < 2$$

independent?


## Example 5

Are the random variables $X$ and $Y$ with joint probability density function

$$f(x, y) = x + y, \quad 0 < x < 1,\ 0 < y < 1$$

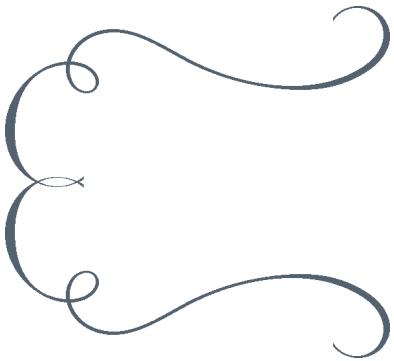
independent?


## Example 6

Let $X_1 \sim \mathrm{Exp}(\lambda_1)$ and $X_2 \sim \mathrm{Exp}(\lambda_2)$ be independent random variables. Find the probability that $X_1 < X_2$

# Thank You





THANK YOU!