Department of Mathematics
College of William & Mary

MATH 451/551

Chapter 6. Joint Distribution

6.2 Independent Random Variables

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Independent Random Variables

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Let the random variables X and Y (discrete or continuous) have a joint distribution described by $f(x, y)$ and marginal distributions described by $f_X(x)$ and $f_Y(y)$. The random variables X and Y are independent iff $\mathbf{f}(\mathbf{x}, \mathbf{y}) = \mathbf{f}_{\mathbf{X}}(\mathbf{x})\mathbf{f}_{\mathbf{Y}}(\mathbf{y})$ for all real numbers x and y.

- Intuitively, if the value oof X does not affect the distribution of Y and if the value of Y does not affect the distribution of X, then X and Y are **independent**.
- Random variables that are not independent are **dependent**.
- An equivalent definition can also be written in terms of cumulative distribution: the random variables X and Y are independent iff $\mathbf{F}(\mathbf{x}, \mathbf{y}) = \mathbf{F}_{\mathbf{X}}(\mathbf{x})\mathbf{F}_{\mathbf{Y}}(\mathbf{y})$ for all real numbers x and y.
- For X and Y to be independent, the support of their joint distribution must be a **product space**, i.e., if X has suppoort $\mathcal{A}$ and Y has support $\mathcal{B}$, then the product space is $\{(\mathbf{x}, \mathbf{y}) | \mathbf{x} \in \mathcal{A} \text{ and } \mathbf{y} \in \mathcal{B}\}$

Example 1

Are the random variables X and Y with joint pmf given below independent?

	1	2	3	$f_X(x)$
1	0.2	0.1	0.3	0.6
2	0.1	0.1	0.2	0.4
$f_Y(y)$	0.3	0.2	0.5	

Example 2

Example 2

Let X_1 and X_2 be random variables with joint pdf

$$f(x_1, x_2) = x_1 x_2, \quad 0 < x_1 < 1,\ 0 < x_2 < 2$$

Are X_1 and X_2 independent?

Thank You

THANK YOU!