Department of Mathematics
College of William & Mary

MATH 451/551

Chapter 6. Joint Distribution

6.1 Bivariate Distribution

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Conditional Distribution

Conditional Distribution

Let the (discrete or continuous) random variables X and Y have a joint distribution described by $f(x, y)$ and marginal distributions described by $f_X(x)$ and $f_Y(y)$. The conditional distribution of X given $Y = y$ is

$$f_{X|Y}(x|Y = y) = \frac{f(x, y)}{f_Y(y)}$$

for $f_Y(y) > 0$, defined over the appropriate support. Likewise, the conditional distribution of Y given $X = x$ is

$$f_{Y|X}(y|X = x) = \frac{f(x, y)}{f_X(x)}$$

for $f_X(x) > 0$, defined over the appropriate support.

- These conditional probability mass functions and probability density functions satisfy the standard existence conditions.

Example 1

Given following joint distribution

$$f(x,y) = \begin{cases} 0.2 & x=1, y=1 \\ 0.1 & x=1, y=2 \\ 0.3 & x=1, y=3 \\ 0.1 & x=2, y=1 \\ 0.1 & x=2, y=2 \\ 0.2 & x=2, y=3 \end{cases},$$

find the probability mass functions associated with the following conditional random variables:

$$f_{X|Y}(x|Y=1) \quad \text{and} \quad f_{Y|X}(y|X=2)$$

Example 2

Deal two cards from a well-shuffled deck. Let the random variable X be the number of aces dealt and let the random variable Y be the number of face cards dealt. Find the conditional distribution of the number of aces in the hand given that there is one face card in the hand.

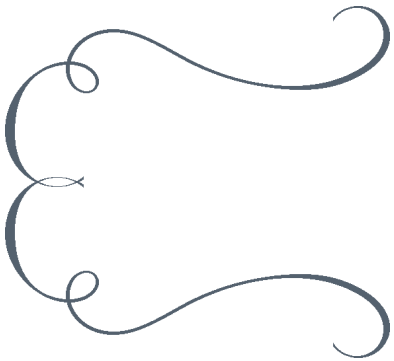
Example 3

Let X and Y have joint pdf

$$f(x,y) = \frac{1}{50}, \quad x > 0, y > 0, x + y < 10$$

1. Find $f_{X|Y}(x|Y = y)$.
2. Find $P(3 < X < 5|Y = 2)$.

Thank You



THANK YOU!