Department of Mathematics
College of William & Mary

MATH 451/551

Chapter 6. Joint Distribution

6.1 Bivariate Distribution

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Marginal Distributions

Marginal Distribution

- For the **discrete random variables** X and Y with joint pmf $f(x, y)$, the marginal pmf's for X and Y are

$$f_X(x) = \sum_y f(x, y) \quad \text{and} \quad f_Y(y) = \sum_x f(x, y)$$

  which are defined over the appropriate supports.

- For the **continuous random variables** X and Y with joint pdf $f(x, y)$, the marginal pdf's for X and Y are

$$f_X(x) = \int_{-\infty}^{\infty} f(x, y)dy \quad \text{and} \quad f_Y(y) = \int_{-\infty}^{\infty} f(x, y)dx$$

  which are defined over the appropriate supports.

Example 1

Let the discrete random variables X and Y have joint probability mass function

$$f(x,y) = \begin{cases} 0.2 & x=1, y=1 \\ 0.1 & x=1, y=2 \\ 0.3 & x=1, y=3 \\ 0.1 & x=2, y=1 \\ 0.1 & x=2, y=2 \\ 0.2 & x=2, y=3 \end{cases}$$

Find the marginal proobability mass functions $f_X(x)$ and $f_Y(y)$.

Example 2

For the continuous random variables X and Y with joint probability density function

$$f(x,y) = \frac{1}{50}, \quad x > 0,\ y > 0,\ x + y < 10,$$

find the probability density function of the marginal distribution of Y.

Example 3

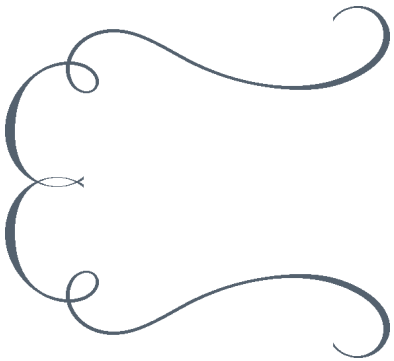
Let the continuous random variables X_1 and X_2 have joint probability density function

$$f(x_1, x_2) = x_1 x_2, \quad 0 < x_1 < 1,\ 0 < x_2 < 2.$$

Find $P(0.2 < X_1 < 0.7)$ by the following three techniques:

1. computing the value of the double integral over the appropriate limits
2. finding the marginal probability ensity function of X_1 and integrating, and
3. finding the marginal cumulative distribution function of X_1 and using the appropriate arguents.

Thank You



THANK YOU!