Department of Mathematics
College of William & Mary

MATH 451/551

Chapter 6. Joint Distribution

6.1 Bivariate Distribution

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Joint Cumulative Distribution Functions

Joint CDF

The joint cumulative distribution function of two random variables (discrete or continuous) X_1 and X_2 is

- $$F(x_1, x_2) = P(X_1 \leq x_1, X_2 \leq x_2).$$

$$\lim_{x_1 \to -\infty} \lim_{x_2 \to -\infty} F_{X_1,X_2}(x_1, x_2) = 0, \lim_{x_1 \to \infty} \lim_{x_2 \to \infty} F_{X_1,X_2}(x_1, x_2) = 1$$

- **Discrete Random Variables:**

$$F(x_1, x_2) = P(X_1 \leq x_1, X_2 \leq x_2) = \sum_{w_1 \leq x_1} \sum_{w_2 \leq x_2} f(w_1, w_2)$$

- **Continuous Random Variables:**

$$F(x_1, x_2) = P(X_1 \leq x_1, X_2 \leq x_2) = \int_{\infty}^{x_1} \int_{\infty}^{x_2} f(w_1, w_2) dw_1 dw_2,$$

and

$$f(x_1, x_2) = \frac{\partial^2 F(x_1, x_2)}{\partial x_1 \partial x_2}.$$

Example 1

Let X_1 and X_2 be continuous random variables with joint probability density function

$$f(x_1, x_2) = x_1 x_2, \quad 0 < x_1 < 1,\ 0 < x_2 < 2.$$

Find the joint cumulative distribution function.

Example 2

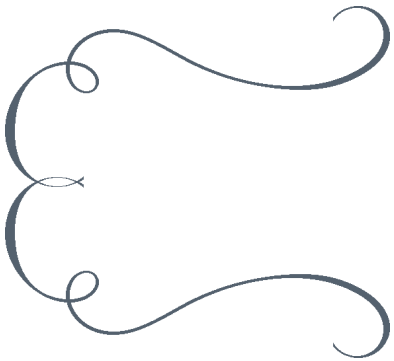
A bag contains 6 red balls, 7 white balls, and 8 blue balls. A random sample of 5 balls is drawn without replacement from the bag. If X_1 denotes the number ooof red balls in the sample and X_2 denotes the number of the white balls in the sample, what is $F(2,3)$?

Univariate Cumulative Distribution

The univariate cumulative distributioon function of one of the variables can be obtained by allowing the argument for the other variable to approach infinity.

$$\begin{aligned} F_X(x) &= P(X \leq x) = P(X \leq x, Y < \infty) \\ &= \lim_{y \to \infty} P(X \leq x, Y \leq y) \\ &= \lim_{y \to \infty} F(x, y) \\ &= F(x, \infty) \end{aligned}$$

Thank You



THANK YOU!