Department of Mathematics
College of William & Mary

MATH 451/551

Chapter 5. Common Continuous Distribution

5.2 Exponential Distribution

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Exponential Distribution

- A continuous random variable with positive support $\mathcal{A} = \{x > 0\}$ is useful in a variety of applications. Examples include
  - patient survival time after the diagosis of a particular cancer
  - the lifetime of a light bulb
  - the waiting and service time for a customer at a coffee shop
  - the time between births at a hospital
  - the number of gallons purchased at a gas pump
  - the time to construct an office building
- A continuous random variable X with pdf

$$f(x) = \lambda e^{-\lambda x}, \qquad x > 0$$

for some real constant $\lambda > 0$ is an Exponential(λ) random variable.

CDF

Memoryless Property

For $X \sim \mathrm{Exp}(\lambda)$ and any two positive real numbers x and y

$$P(X \geq x + y | X \geq x) = P(X \geq y).$$

Theorem

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If $X \sim \mathrm{Exp}(\lambda)$, then

$$E\left(X^s\right) = \frac{\Gamma(s+1)}{\lambda^s}, \quad s > -1$$

where $\Gamma(k) = \int_0^\infty x^{k-1} e^{-x} dx$.

Notes on Gamma function

- When k is an integer, $\Gamma(k) = (k-1)!$
- The gamma function is minimized at $k \cong 1.4616$
- $\Gamma(k+1) = k\Gamma(k)$ fpr $k > 0$
- $\Gamma\left(\frac{1}{2}\right) = \sqrt(\pi)$
- The gamma function is calculated in R with *gamma*
- The gamma function is calculated in Maple with *GAMMA*
- $E\left(X^s\right) = \frac{s!}{\lambda^s}$

Mean

Variance

MGF

Skewness

Kurtosis

R Functions

R Functions

Function	Returned Value
dexp(x, λ)	calculates the probability density function $f(x)$
pexp(x, λ)	calculates the cumulative distribution function $F(x)$
qexp(u, λ)	calculates the percentile (quantile) $F^{-1}(u)$
rexp(m, λ)	generates m random variates

Example 1

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Let $X \sim \mathrm{Exp}(\lambda)$, what is the distribution of $Y = \lfloor X \rfloor$?

Summary

The exponential distribution

- is the fundamental distribution with positive support
- has a single positive parameter λ
- is the only continuous distribution with the memoryless property
- has pdf

$$f(x) = \lambda e^{-\lambda x}, \quad x > 0$$

- has moments

$$\mu = \frac{1}{\lambda} \quad \text{and} \quad \sigma^2 = \frac{1}{\lambda^2}$$

- has a second parameterization: $\theta = \frac{1}{\lambda}$

$$f(x) = \frac{1}{\theta} e^{\frac{-x}{\theta}}, \quad x > 0$$

and the population mean and variance are

$$\mu = \theta \quad \text{and} \quad \sigma^2 = \theta^2$$

THANK YOU!