Department of Mathematics
College of William & Mary

MATH 451/551

Chapter 4. Common Discrete Distributions

4.3 Geometric Distribution

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Mean

Variance

MGF

Skewness

Kurtosis

Memoryless Property

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For $X \sim Geo(p)$ and any two nonnegative integers x and y,

$$P(X \geq x + y | X \geq x) = P(X \geq y).$$

Interpretation

Consider a sequence of repeated, mutually independent, and identically distributed Bernoulli trials and a random variable X that is the number of failures before the first success. If you know that X is greater than or equal to x, then the distribution of the **remaining** number of Bernoulli trials before the first success has the same distributioon as if the original x trials had never occurred. This interpretation is consistent with intuition: the previous history of the sequence of Bernoulli trials has no effect on the outcomes of future Bernoulli trials.

Example 1

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Roll a pair of fair dice repeatedly until a "double six" appears. Let X be the number of rolls prior to the appearance of the first double six. Find $f(x)$, $E(X)$, $V(X)$, and $P(X < 24)$.

R Functions

Function	Returned Value
dgeom(x, p)	calculates the probability mass function $f(x)$
pgeom(x, p)	calculates the cumulative mass function $F(x)$
qgeom(u, p)	calculates the percentile (quantile) $F^{-1}(u)$
rgeom(m, p)	generates m random variates

Alternative Definition

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The geometric distribution can also be parameterized as the trial number of the first success.

- **PMF:** A discrete random variable X with PMF

$$f(x) = p(1-p)^{x-1}, \qquad x = 1, 2, \dots$$

  for $0 < p < 1$ is a *Geometric*(p) random variable.
- This shifts the distribution one unit to the right.

$$\mu = \frac{1}{p} \quad \text{and} \quad \sigma^2 = \frac{1-p}{p^2}.$$

Example 2

How many tosses of a pair of fair dice are necessary to be 99% certain that a double six will appear?

THANK YOU!