Department of Mathematics
College of William & Mary

MATH 451/551

Chapter 4. Common Discrete Distributions

4.2 Binomial Distribution

GuanNan Wang
gwang01@wm.edu

Binomial Distribution

Binomial Distribution

- When n Bernoulli trials are conducted, each with an identical probability of success p, the experiment is known as a **binomial** random experiment.
- A binomial random experiment satisfies the following criteria.
  1. The random experiment consists of n identical trials
  2. There are two possible outcomes for each trial
  3. The trials are mutually independent
  4. The probability of success on each trial is identical
- $X \sim Binomial(n, p)$ models the number of successes in n mutually independent Bernoulli trials, each with probability of success p, where n is a positive integer.
- **Support of X:** $\mathcal{A} = \{0, 1, 2, \ldots, n\}$.
- **PMF:** $f(x) = \binom{n}{x} p^x (1-p)^{n-x}, \; x = 0, 1, 2, \ldots, n$ for some positive integer n and $0 < p < 1$ is a $Binomial(n, p)$ random variable.

Mean

Variance

MGF

Skewness

Kurtosis

Example 1

If Emma takes 3 free throws (she is a 70% free throw shooter) and X is the number that she makes, find $f(x)$, μ, σ^2, and $P(X = 2)$.

Example 2

A dozen eggs contain 3 defectives. If a sample of 5 is taken with replacement, find the probability that

1. exactly 2 of the eggs sampled are defective
2. 2 or fewer of the eggs sampled are defective

Example 3

An airplane has 100 seats. The airline “overbooks” a flight (sells more tickets than available seats) in order to maximize their profit. Assume that each ticket holder’s decision to show up for a flight is an independent Bernoulli trial with a probability of showing up for the flight of 0.92. If the airline profit is $10 for each seat sold and the airline loses $40 for each “bumped” passenger, what is the expected profit if 103 seats are sold?

Example 4

A company produces biased coins that come up heads when flipped with probability 0.7. You are not sure whether you have one of these biased coins or whether you have a fair coin, so you devise the following experiment: (1) Flip the coin 100 times, (2) If there are 62 or more heads conclude that the coin is biased, otherwise, conclude that the coin is fair.

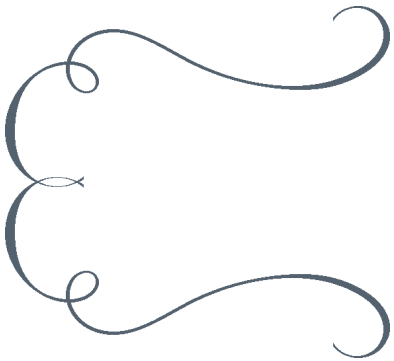
		True Coin Status	
		Coin fair	Coin biased
Experiment Conclusion	Coin fair		
	Coin biased		

R Functions

R Functions

Function	Returned Value
dbinom(x, n, p)	calculates the probability mass function $f(x)$
pbinom(x, n, p)	calculates the cumulative mass function $F(x)$
qbinom(u, n, p)	calculates the percentile (quantile) $F^{-1}(u)$
rbinom(m, n, p)	generates m random variates

Thank You



THANK YOU!