Department of Mathematics
College of William & Mary

MATH 451/551

Chapter 4. Common Discrete Distributions

4.1 Bernoulli Distribution

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Examples & Notations

Examples

- **Bernoulli distribution** arises in clinical trials, athletics, polling
- **Poisson distribution** arises in radioactive decay, arrival processes, quality control
- **Normal distribution** arises in agricultural yields, IQ scores, heights of children

Notations

- the support is $\mathcal{A}$
- the parameter space is Ω
- the probability mass function is $f(x) = P(X = x)$
- the cumulative distribution function is $F(x) = P(X \leq x)$
- the moment generating function is $M(t) = E\left(e^{tX}\right)$
- the population mean is $E(X) = \mu$
- the population variance is $V(X) = \sigma^2$

Bernoulli Distribution

Bernoulli Distribution

- The **Bernoulli distribution** models a **binary** random variable X with support $\mathcal{A} = \{x|x = 0, 1\}$.
- A discrete random variable X with probability mass function
$$f(x) = \begin{cases} 1-p & x = 0 \\ p & x = 1 \end{cases}$$
for $0 < p < 1$ is a Bernoulli (p), $Ber(p)$ random variable.
- **Parameter Space:** $\Omega = \{p|0 < p < 1\}$.
- **Compact way to write** $f(x)$**:** $f(x) = p^x(1-p)^{1-x}, \quad x = 0, 1.$
- **Bernoulli Trial:** A single observation of a Bernoulli random variable.
- **Outcomes of a Bernoulli trial are generically called:**
  - success when $X = 1$
  - failure when $X = 0$

Mean

Variance

MGF

Skewness

Kurtosis

Remarks

- Shorthand: $X \sim Bernoulli(p)$ or $X \sim Ber(p)$.
- Important special case when $p = \frac{1}{2}$ (fair coin toss)
  - population mean is $\mu = \frac{1}{2}$
  - population variance is $\sigma^2 = \frac{1}{4}$ (largest possible)
  - population skewness is 0 (symmetric)
  - population kurtosis is 1 when $p = \frac{1}{2}$ (the smallest population kurtosis possible for any random variable)
- Repeated Bernoulli trials form the basis for next three distributions
  - binomial
  - geometric
  - negative binomial

THANK YOU!