Department of Mathematics
College of William & Mary

MATH 451/551

Chapter 3. Random Variables

3.5 Inequalities

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Markov's Inequality

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Let X be a random variable with nonnegative support (that is, $P(X \geq 0) = 1$) for which $E(X)$ exists. Then for any positive constant a

$$P(X \geq a) \leq \frac{E(X)}{a}.$$

Example 1

Example 1

Roll a fair die 60 times. Let the random variable X be the number of sixes that appear. Use Markov's inequality to find an upper bound on the probability of rolling 30 or more sixes.

Markov's Inequality Extension

Let $g(X)$ be a nonnegative function of the random variable X. If $E\{g(X)\}$ exists, then, for every positive, real constant a,

$$P\{g(X) \geq a\} \leq \frac{E\{g(X)\}}{a}.$$

Chebyshev's Inequality

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Let the random variable X have a finite population mean μ and a finite population variance σ^2. For every real-valued $k > 0$

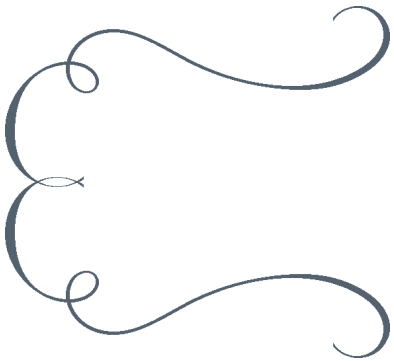
$$P\{|X - \mu| \geq k\sigma\} \leq \frac{1}{k^2}.$$

Example 2

Example 2

Let X be the number of screws delivered to a box by an automatic filling device. Assume $\mu = 1000$ and $\sigma^2 = 25$. There are problems associated with having too many (giving away free product) or too few (potential irritated customers) screws in a box. Use Chebyshev's inequality to find a bound on $P(994 < X < 1006)$.

Thank You



THANK YOU!