Department of Mathematics
College of William & Mary

MATH 451/551

Chapter 3. Random Variables

3.4 Moment Generating Function

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Moment Generating Functions

Moment Generate Function (MGF)

Let X be a random variable, then the **moment generating function** (MGF) of X is

$$M(t) = E\left(e^{tX}\right)$$

provided that the expected value exists on the interval $-h < t < h$ for some positive real number.

- The moment generating function at $t = 0$ must be 1, i.e. $M(0) = E\left(e^{0}\right) = 1$.

- they are good at generating moments
- they will also be used to find the distribution of sums of independent random variables
- they will also be used to find the limiting distribution of a random variable

Example 18

Example 18

Find the moment generating function for a continuous random variable X that is uniformly distributed between 0 and 1.

Example 19

Example 19

Consider the random variable X with moment generating function

$$M(t) = 0.7e^{t} + 0.2e^{2t} + 0.1e^{3t}, \quad -\infty < t < \infty.$$

Is X discrete or continuous? What is the probability mass function or probability density function of X?

Moment Generating Functions

Theorem 3.8

If X has moment generating function $M(t)$ then for some positive integer r

$$E\left(X^r\right) = M^{(r)}(0) = \frac{d^r}{dt^r} M(t)\bigg|_{t=0}.$$

provided that the expected value exists on the interval $-h < t < h$ for some positive real number.

Example 20

Example 20

Use the moment generating function to find $E(X)$, $E\left(X^2\right)$, and $E\left(X^3\right)$ for the continuous random variable X with probability density function

$$f(x) = e^{-x}, \qquad x > 0.$$

Example 21

Use the moment generating function to find $E(X)$, $E\left(X^2\right)$, and $E\left(X^3\right)$ for the discrete random variable X with probability mass function

$$f(x) = \begin{cases} 0.7, & x = 1 \\ 0.2, & x = 2 \\ 0.1, & x = 3 \end{cases}.$$

Example 22

Consider the random variable $X \sim N(\mu, \sigma^2)$, with probability density function

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}, \qquad -\infty < x < \infty.$$

Find the distribution for $Y = 3X + 4$.

Example 23

Let X be a random variable with pdf

$$f(x) = \begin{cases} e^{1-x} & x > 1 \\ 0 & \text{o.w.} \end{cases}.$$

- Find the moment generating function (MGF) of X.
- Find $E(X)$ and $V(X)$.
- Suppose that $X_1, X_2, \ldots, X_{2n-1}, X_{2n}$ are independent random variables with PDF $f(x)$, as given above. Let $T_n = \sum_{i=1}^{2n} (-1)^{i-1} X_i$, find the moment generating function of T_n.

THANK YOU!