Department of Mathematics
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MATH 451/551

Chapter 3. Random Variables

3.4 Moments

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Population Moment

Population Moment

For a random variable X and positive integer r, the rth population moment X about the origin is $E(X^r)$ when the expectation exists.

Central Population Moment

For a random variable X and positive integer r, the rth population moment X about the population mean is $E\{(X-\mu)^r\}$ when the expectation exists.

Standardized Random Variable

Standardized Random Variable

For a random variable X with population mean μ and positive population standard deviation σ, the random variable $\frac{X-\mu}{\sigma}$ is called a **standardized random variable**, which has population mean 0 and population standard deviation 1 when the expectations exist.

Population Skewness

Population Skewness

For a random variable X with population mean μ and positive population standard deviation σ, the population skewness $E\left\{\left(\frac{X-\mu}{\sigma}\right)^3\right\}$ when the expectation exists.

- The population skewness is a measure of the symmetry of a probability distribution.
- If $f(x)$ is symmetric, the population skewness is 0.
- If $f(x)$ is skewed to the right, the population skewness is positive.
- If $f(x)$ is skewed to the left, the population skewness is negative.

Population Kurtosis

Population Kurtosis

For a random variable X with population mean μ and positive population standard deviation σ, the population kurtosis $E\left\{\left(\frac{X-\mu}{\sigma}\right)^4\right\}$ when the expectation exists.

- The population kurtosis is a measure of the heaviness of the tails of a probability distribution.

Example 16

Find the population skewness and kurtosis of a continuous random variable X that is uniformly distributed between 0 and 1.

Example 17

Find the population skewness and kurtosis of a continuous random variable X with probability density function

$$f(x) = \frac{x}{2}, \quad 0 < x < 2.$$

THANK YOU!