Department of Mathematics
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MATH 451/551

Chapter 3. Random Variables

3.4 Variance

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Moments

Variance

For a random variable X with population mean μ, the **population variance** of X is $\sigma^2 = V(X) = E\{(X - \mu)^2\}$ when the expected values exist.

- The units on the population variance are square of the units of the random variable X.
- The positive square root of the population variance is the **population standard deviation** σ. One reason for the popularity of σ is that its units are the same units as the random variable X.
- If the discrete random variable X has a support $\mathcal{A}$ that contains only one x-value, then $\sigma^2 = 0$. This distribution is often known as a **degenerate** distribution.
- Some authors use $V(X)$ for the population variance.
- The population variance is not the only measure of dispersion for a random variable X.
    - the population range $R = \sup(\mathcal{A}) - \inf(\mathcal{A})$,
    - the population mean absolute deviation $E(|X - \mu|)$.

Moments

Variance

For the random variable X with population mean μ and population variance σ^2, $V(X) = E(X^2) - \mu^2$, when is known as the **shortcut formula** for computing the population variance.

Example 11

Using the defining and computational formulas, find the population variance of the number of spots showing when rolling a fair die.

Example 12

Example 12

Calculate the population variance of the random variable X with probability density function

$$f(x) = \frac{x}{2}, \quad 0 < x < 2$$

using the defining formula and the computation formula.

Properties of Variance

Property 7

For the random variable X with population mean μ and population variance σ^2,

$$V(aX + b) = a^2 V(X)$$

for real constants a and b.

Example 13

Random variable X has the probability density function

$$f(x) = \begin{cases} \frac{3}{2}x^2 + x, & 0 \leq x \leq 1 \\ 0, & \text{o.w.} \end{cases}.$$

Find $V(X)$.

Example 14

Consider the random variable X with cumulative distribution function

$$F(x) = \begin{cases} 0, & x < -1 \\ \frac{x}{5} + \frac{3}{10}, & -1 \leq x < 0 \\ \frac{2}{5}, & 0 \leq x < 1 \\ \frac{x}{5} + \frac{2}{5}, & 1 \leq x < 3 \\ 1, & x \geq 3 \end{cases}.$$

Find $E(X)$ and $V(X)$.

Example 15

Let Y be a random variable of the continuous type with PDF $f(y)$, which is positive provided $0 < y < b < 1$, and is equal to zero elsewhere. Show that

$$E(Y) = \int_0^b \{1 - F(y)\} dy,$$

where $F(y)$ is the cumulative distributioon function of Y.

THANK YOU!