Department of Mathematics
College of William & Mary

MATH 451/551

Chapter 3. Random Variables

3.4 Properties of Expected Values

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Properties of Expected Values

Property 1

Let X be a random variable defined on the support $\mathcal{A}$ with probability mass function $f(x)$ if X is discrete and probability density function $f(x)$ if X is continuous. The expected value of $g(X)$ is

$$\mathrm{E}\{g(X)\} = \begin{cases} \sum_{\mathcal{A}} g(x)f(x) & X \text{ is discrete} \\ \int_{\mathcal{A}} g(x)f(x)dx & X \text{ is continuous} \end{cases}$$

when the sum of integral exists. When the sum or integral diverges, the expected value is undefined.

Properties of Expected Values

Property 2

Given a random variable X and a real constant c

$$\mathrm{E}(c) = c.$$

Properties of Expected Values

Property 3

Given a random variable X and a real constant c

$$\mathrm{E}(cX) = c\mathrm{E}(X).$$

Example 9

Example 9

Let the continuous random variable X be uniformly distributed between 0 and 1 with probability density function

$$f_X(x) = 1, \quad 0 < x < 1.$$

Find $E\left(\sqrt{X}\right)$.

Example 10

Consider the discrete random variable X with probability mass function

$$f_X(x) = \frac{1}{3}, \quad x = -2, 0, 1.$$

Find $E\left(|X| + 4\right)$.

Properties of Expected Values

Property 4

Given a random variable X, a real constant c, andd a functioon $g(X)$,

$$\mathrm{E}\{cg(X)\} = c\mathrm{E}\{g(X)\}.$$

when the expected values exist.

Properties of Expected Values

Property 5

Give a random variable X and functions $g_1(X)$ and $g_2(X)$,

$$\mathrm{E}\{g_1(X) + g_2(X)\} = \mathrm{E}\{g_1(X)\} + \mathrm{E}\{g_2(X)\}.$$

when the expected values exist.

Property 5 (extension)

Give a random variable X and functions $g_1(X), g_2(X), \ldots, g_k(X)$

$$\mathrm{E}\{g_1(X)+g_2(X)+\cdots+g_k(X)\} = \mathrm{E}\{g_1(X)\}+\mathrm{E}\{g_2(X)\}+\cdots+\mathrm{E}\{g_k(X)\}.$$

Thank You

THANK YOU!