Department of Mathematics
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MATH 451/551

Chapter 3. Random Variables
3.4 Expected Values

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Expected Values

- The mathematical expectation, or expected value, of a random variable X is what you "expect" (on the average, in the long run) for the value of X.
- The standard notation for the expected value of X is $\mu = \mathrm{E}(X)$.
- The expected value of X is also known as the "population mean".

Expected Value

Let X be a random variable defined on the support $\mathcal{A}$ with probability mass function $f(x)$ if X is discrete or probability density function $f(x)$ if X is continuous.

$$\mu = E(X) = \begin{cases} \sum_{\mathcal{A}} xf(x) & X \text{ is discrete} \\ \int_{\mathcal{A}} xf(x)dx & X \text{ is continuous} \end{cases}$$

when the sum or integral exists. When the sum or integral diverges, the expected value is undefined.

Example 1

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Example 1

Find the expected number of spots (pips!) showing when rolling a fair die.

Example 2

Example 2

The probability mass function foor the random variable X is
$$f(x) = \frac{7^x e^{-7}}{x!}, \quad x = 0, 1, 2, \ldots.$$
What is the expected value of X?

Example 3

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Example 3

An urn contains 3 red balls and 4 blue balls. Balls are drawn successively at random and without replacement from the urn. Let the random variable X be the trial number when the first red ball is drawn. Find $E(X)$.

Example 4

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Example 4

Let n be a positive integer. A cube is comprised of n^3 smaller cubes. If one of the n^3 smaller cubes is selected at random, given an expression for the expected number of exposed faces.

THANK YOU!