Department of Mathematics
College of William & Mary

MATH 451/551

Chapter 3. Random Variables

3.3 Transformations

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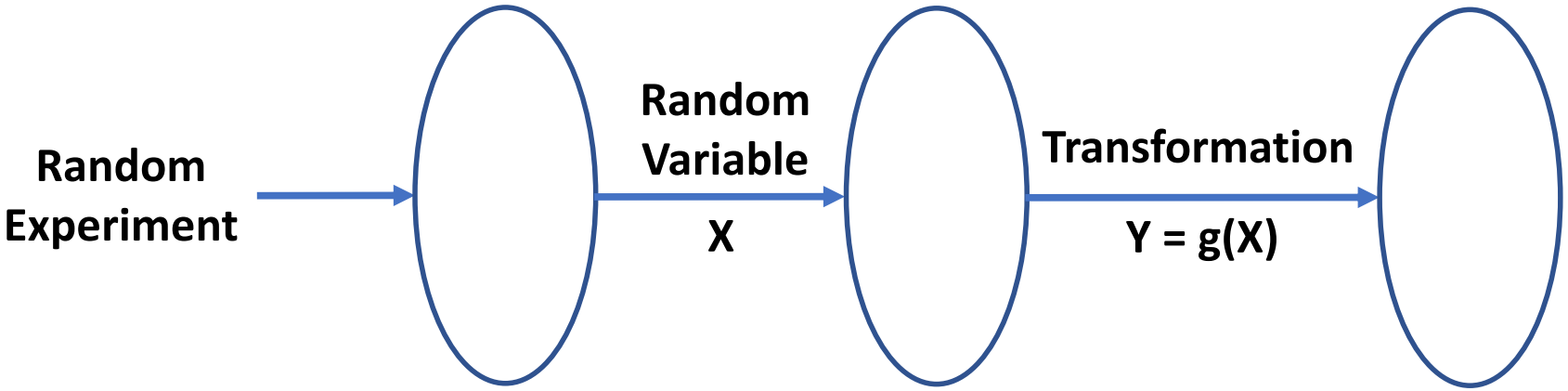
Transformations of Random Variables



- $F_X(x)$: cumulative distribution function for a random variable X;
- $F_Y(y)$: cumulative distribution function for a random variable Y;
- $f_X(x)$: probability mass/density function for a random variable X;
- $f_Y(y)$: probability mass/density function for a random variable Y.

$$F_Y(y) = P(Y \leq y) = P\{g(X) \leq y\}.$$

Note

- We are often interested in finding the distribution of a function of a random variable.
- Our goal here is to find the cumulative distribution function of Y.

Example 8

Let the random variable X be uniformly distributed between 0 and 1. Find the cumulative distribution function of $Y = g(X) = \sqrt{X}$.

$$F_X(x) = \begin{cases} 0, & x \leq 0 \\ x, & 0 < x < 1 \\ 1, & x \geq 1 \end{cases}$$

Example 9

Let the random variable X be uniformly distributed between 0 and 1. Find the probability density function of $Y = g(X) = \arcsin(X)$.

$$F_X(x) = \begin{cases} 0, & x \leq 0 \\ x, & 0 < x < 1 \\ 1, & x \geq 1 \end{cases}$$

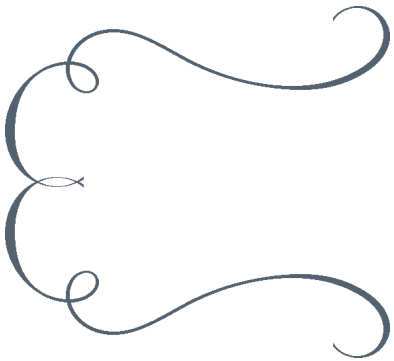
Example 10

Let the random variable X have the probability density function

$$f_X(x) = \exp(-x) = e^{-x}, \quad x > 0.$$

Find the cumulative distribution function of $Y = g(X) = X^2$.

Thank You



THANK YOU!