Department of Mathematics
College of William & Mary

MATH 451/551

Chapter 3. Random Variables

3.2 Continuous Random Variables

GuanNan Wang
gwang01@wm.edu

Motivating Example

Example 1

What probability distribution formalizes the notion of “equally-likely" outcomes in unit interval $[0, 1]$?

Continuous Random Variable

Continuous Random Variable

A **continuous random variable** X has a support set $\mathcal{A}$ that is uncountable.

Probability Density Function (PDF)

Probability density function (pdf) existence conditions:

- $0 \leq f(x),\ x \in \mathcal{A}$.
- $\int_{\mathcal{A}} f(x)dx = 1$.
- For $A \subset \mathcal{A}$, $P(X \in A) = \int_A f(x)dx$.
- For real constants $a < b$, $P(a < X < b) = \int_a^b f(x)dx$.
- Endpoints don't matter:

  $$P(a < X < b) = P(a \leq X < b) = P(a < X \leq b) = P(a \leq X \leq b),$$

  and for any value a, $P(X = a) = 0$.

Example 2

Let the continuous random variable X have pdf

$$f(x) = \frac{x}{2}, \quad 0 < x < 2.$$

Find the probability that X is greater than 1.

Example

Example 3

Let the continuous random variable X have probability density function

$$f(x) = e^{-x}, \quad x > 0$$

Find the probability that $\lfloor x \rfloor$ is even. The floor of X is even is equivalent to X falling in one of these intervals
$[0, 1),\ [2, 3),\ [4, 5),\ \cdots.$

THANK YOU!