

MATH 451/551

Chapter 3. Random Variables

3.1 Discrete Random Variables

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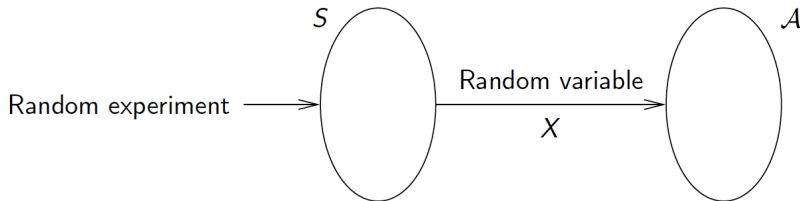


Random Variables



Given a random experiment with an associated sample space S , a **random variable** is a function X that assigns to each element $s \in S$ one and only one real number $X(s) = x$. The **support** of X is the set of real numbers $\mathcal{A} = \{x \mid x = X(s), s \in S\}$.

A fair coin is tossed twice. Find the probability that both tosses come up heads.





Discrete Random Variable

Let X denote a random variable with one-dimensional support \mathcal{A} , then a random variable X is discrete if the support \mathcal{A} is countable, that is

- ▶ X is a discrete random variable if \mathcal{A} is a finite set, or
- ▶ X is a discrete random variable if \mathcal{A} is a denumerable set.

Probability Mass Function

- ▶ The probability that X takes on the value x , $P(X = x)$, is defined as the sum of the probabilities of all sample points in S that are assigned the value x . Sometimes, we denote $P(X = x) = p(x)$.
- ▶ The probability distribution for a discrete random variable (r.v.) X can be represented by a formula, a table, or a graph which provides $P(X = x) = p(x)$ for all x , such that
 - ▶ $0 \leq p(x) \leq 1, \forall x$
 - ▶ $\sum_x p(x) = 1$, where \sum_x is the sum over all possible values, x , of the r.v. x

Example



Example 1

Toss a fair coin twice times. Find the probability distribution for the number of heads.

Example



Example 2

A 5-card hand is dealt from a well-shuffled 52-card deck. Let the random variable X be the number of jacks in the hand. Find the support of X and the probability mass function of X .

Example



Example 3

A box contains 6 green balls and 10 red balls. If we draw 2 balls without replacement from the box, let $X = \#$ of green balls drawn, find the probability mass function of X .

Example



Example 4

A spinner yields three equally-likely outcomes: 1, 2, 3. If the random variable X denotes the product of the outcomes of two spins, find the probability mass function $f(x)$, $P(X = 6)$, and $P(X \leq 6)$.

Example



Example 5

Flip a fair coin repeatedly until a head appears. Let X be the number of flips required. Find the probability mass function of X and find $P(X \geq 4)$.

Thank You



THANK YOU!