Department of Mathematics
College of William & Mary

MATH 451/551

Chapter 2. Probability

2.6 Independent Events

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Independent Events

Events A and B are **independent** if and only if

$$P(A \cap B) = P(A)P(B).$$

The following four statements are equivalent:

- A and B are independent events,
- $P(A \cap B) = P(A)P(B)$,
- $P(A|B) = P(A)$,
- $P(B|A) = P(B)$.

Remarks: The last two statements capture the essence of the independence of two events: the occurrence (or nonoccurrence) of one event doesn't affect the probability of another event occurring. Events that are not independent are said to be **dependent**.

Example 1

A single card is drawn at random from a 52-card deck. Let the event H be that the suit of the card is hearts. Let the event Q be that the rank of the card is a queen. Are the event H and Q independent?

Example 2

A fair coin is tossed twice. Show that the events

A: the first toss yields heads,

B: the second toss yields heads,

C: the two tosses yield different results,

are pairwise independent, but $P(A \cap B \cap C) \neq P(A)P(B)P(C)$.

Mutually Independent

Events $A_1, A_2, \ldots, A_n$ are **mutually independent** if and only if the probability of occurrence of the intersection of any 2, 3, . . ., or n of these events is equal to the product of their associated probabilities of occurrence.

If the case of three events, A_1, A_2 and A_3, the following equations must be satisfied for three events to be mutually independent:

$$\begin{aligned} P(A_1 \cap A_2) &= P(A_1)P(A_2) \\ P(A_1 \cap A_3) &= P(A_1)P(A_3) \\ P(A_2 \cap A_3) &= P(A_2)P(A_3) \\ P(A_1 \cap A_2 \cap A_3) &= P(A_1)P(A_2)P(A_3) \end{aligned}$$

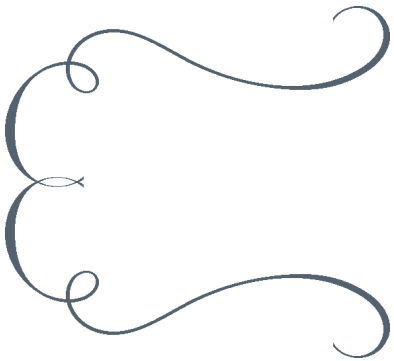
Example 1

Consider a series system of n mutually independent components with probabilities of functioning $p_1, p_2, \ldots, p_n$. If all components must function for the system to function, find the probability that the system functions.

Example 2

Consider a parallel system of n mutually independent components with probabilities of function, find the probability that the system functions.

Thank You



THANK YOU!