Department of Mathematics
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MATH 451/551

Chapter 2. Probability

2.5 Rule of Bayes

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Rule of Bayes

Let $A_1, A_2, \ldots, A_n$ be a set of events that partition the sample space S, and $P(A_i) > 0$ for $i = 1, 2, \ldots, n$. For any event B with $P(B) > 0$,

$$P(A_j|B) = \frac{P(B|A_j)P(A_j)}{\sum_{i=1}^{n} P(B|A_i)P(A_i)}$$

$j = 1, 2, \ldots, n$.

Example 1

Moe, Curly and Larry are gas station attendants. Moe handles 30% of the customers, Curly handles 50% of the customers, and Larry handles 20% of the customers. They are always supposed to wash the customer's windshield. Moe forgets 1 time in 20, Curly forgets 1 time in 10, and Larry forgets 1 time in 2.

1. What is the probability that a windshield has NOT been washed?
2. Given that a windshield was not washed, what is the probability that it was Curly who didn't wash it?

Example 2

The “car and goats" problem, also known as the “Monty Hall" paradox or the “Let’s Make a Deal" problem, can be solved using the rule of Bayes. The game show host, Monty Hall, shows you three closed doors. There is a car behind one of the doors and goats behind the other two. If you open the door with the car behind it, you keep the car. You select a door, but before the door is opened, Monty Hall opens one of the other doors to reveal a goat, then gives you the option of switching doors. Is there any advantage to switching?

THANK YOU!