Department of Mathematics
College of William & Mary

MATH 451/551

Chapter 1. Introduction

1.3.1 Sets

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Sets

Set

A **set** is a collection of objects (elements), usually denoted by capital letters such as $A, B, \cdots$.

$$\begin{aligned} A &= \{1, 2, \ldots, 1000\} \\ B &= \{x | x \text{ is a positive integer less than } 101\} \\ C &= \{\text{Bulls, Trailblazers}\} \\ D &= \{(x, y) | 0 < x < 1, 0 < y < 2\} \end{aligned}$$

Elements

If an object belongs to a set, it is said to be an **element** of the set. The notation $\in$ is used to denote membership in a set.

Using the sets defined in the previous example,

$$17 \in A \quad 99 \in B \quad \left(\frac{2}{3}, 1\right) \in D \quad Cubs \notin C$$

Set Operations

Subset

If every element of the set A is also an element of the set B, then A is a **subset** of B. The notation $\subset$ is used to denote the subset relationship.

$$A \subset B \text{ iff } \forall x \in A \Rightarrow x \in B$$

The natural numbers N, also known as the positi-ve integers, are a subset of the integers Z, which are a subset of the rational numbers Q, which are a subset of the real numbers R, which are a subset of the complex numbers C. These relationships are compactly stated as

$$N \subset Z \subset Q \subset R \subset C$$

Venn Diagram

Venn Diagram

A **Venn Diagram** is a useful tool in set theory and in probability for sorting out the relationships between sets.

- The external rectangle that is drawn outside of the two sets A and B is called the **universal set**, and it contains all possible elements under consideration.

When there are several subsets involved in a particular application, we often use subscripts, rather than individual letters to denote the sets. Use Venn Diagram to present the relationship between

$$A_1 = \{x|0 < x < 1\} \quad \text{and} \quad A_2 = \{x|0 < x < 5\}.$$

Set Operations (Cont.)

Equality

$$A = B \text{ iff } A \subset B \text{ and } B \subset A.$$

Null Set

A set containing no elements is called the **null set** (also known as the **empty set**. The notation Φ is used to denote the null set.)

Set Operations (Cont.)

Union

The union of A and B, $A \cup B$, is the set of all elements (points) that belongs to either A or B or *both*.

$$A \cup B = \{x | x \in A \text{ or } x \in B\}.$$

$A_1 \cup A_2 \cup \ldots = \cup_{i=1}^{\infty} A_i = \{x | x \in A_1 \text{ or } x \in A_2 \text{ or } \ldots\}$, which applies to a finite or infinite number of sets.

Example 1

For the sets $A_1 = \{x|0 < x < 1\}$ and $A_2 = \{x|0 < x < 5\}$, find the union of A_1 and A_2.

Let $A_k = \{k, k+1, k+2, \ldots, k^2\}$ for $k = 1, 2, \ldots$. Find the union of A_3, A_4, and A_5.

Set Operations (Cont.)

Intersection

The intersection of A and B, $A \cap B$, is the set of all elements (points) that belongs to both A and B.

$$A \cap B = \{x | x \in A \text{ and } x \in B\}.$$

$A_1 \cap A_2 \cap \ldots = \cap_{i=1}^{\infty} A_i = \{x | x \in A_1,\ x \in A_2,\ \ldots\}$, which applies to a finite or infinite number of sets.

Example 2

For the sets $A_1 = \{(x, y)|x^2 + y^2 \leq 16\}$ and $A_2 = \{(0, 0), (1, 1), (2, 4), (3, 9), \cdots\}$, find the intersection of A_1 and A_2.

Disjoint (Mutually Exclusive)

Disjoint (Mutually Exclusive)

If $A \cap B = \Phi$, then A and B are **disjoint** or **mutually exclusive**.

Set Operations

Complement

The **compliment** of A, denoted $\bar{A}$ or A^c, is the set of all elements (points) not in A.

$$A^c = \{x | x \notin A\}.$$

Example 3

Let the set A be the set of all real numbers on the open interval $(0, 1)$, that is $A = \{x|0 < x < 1\}$. Find A^c.

Example 4

Consider an experiment of selecting a card at random fro a standard deck and noting its suit: clubs (C), diamonds (D), hearts (H), or spades(S). All the possible outcomes are $S = \{C, D, H, S\}$. Let $A = \{C, D\}$ and $B = \{D, H, S\}$, find the following sets:

1. $A \cup B$

2. $A \cap B$

3. A^c

4. $(A \cup B)^c$

Thank You

THANK YOU!