Department of Mathematics
College of William & Mary

MATH 451/551

Chapter 1. Introduction
1.2 Counting

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What is Statistics?

- *Webster's New Collegiate Dictionary:* "A branch of mathematics dealing with the collection, analysis, interpretation, and presentation of masses of numerical data."

- *Stuart and Ord (1991):* "Statistics is the branch of the scientific method which deals with the data obtained by counting or measuring the properties of population."

- *Rice (1995):* "Statistics is essentially concerned with procedures for analyzing data, especially data that in some vague sense have a random character."

- *Freund and Walpole (1987):* "Statistics is the science of basing inferences on observed data and the entire problem of making decisions in the face of uncertainty."

All the authors imply that *"The objective of statistics is to make an inference about a population based on information contained in a sample from that population and the provide an associated measure of goodness for the inference."*

Enumeration

Enumeration involves listing all of the possible outcomes to an experimemt.

The Chicago Cubs and the Chicago White Sox are playing in the World Series. The best-of-seven series is tied at two games apiece. What are the possible outcomes to the series?
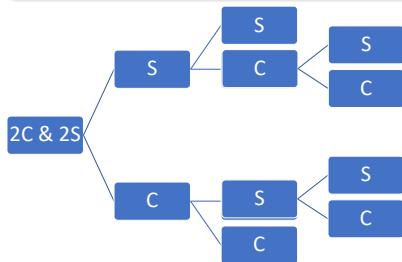
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Multiplication Rule

Assume that there are r decisions to be made. If there are n_1 ways to make decision 1, n_2 ways to make decision 2, ..., n_r ways to make decision r, then there are $n_1 \times n_2 \times \cdots \times n_r$ ways to make all decisions.

How many different sequences of heads and tails are possible in 16 tosses of a fair coin?

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Example 1

4

We go to Chipole to have lunch. To make an order, the first step is to choose your type of lunch, including **Burrito**, **Bowl**, **Salad**, and **Taco**. In the second step, we need to choose the protein/veggie, including **Chicken**, **Steak**, **Barbacoa**, **Carnitas**, **Sofritas**, and **Veggie**. In the third step, we can choose from **White Rice**, **Brown Rice**, and **No Rice**. Finally, we can choose from **Black Beans**, **Pinto Beans**, and **No Beans**. How many different ways we can build up our main dish at Chipole?

Example 2

5

1. How many ways can a family of 5 line up for photograph?

2. How many ways can a family of 5 that consists of 3 men and 2 women line up for a photograph so that men and women alternate?

Other Examples

1. How many ways can a mother give away 8 dogs to her 3 children?

2. How many ways are there to arrange the letters in the world "dynamite"?

3. How many ways can a family of 5 that consists of 3 men and 2 women line up for a photograph so that men and women alternate?

Permutations

Permutation

A **permutation** is an ordered arrangement of r objects selected from a set of n distinct objects without replacememt.

List the permutations from the set $\{a, b, c, d\}$ selected 2 at a time.

The number of permutations of n distinct object selected r at a time without replacement is

$$n \times (n-1) \times (n-2) \times \cdots \times (n-r+1) = \frac{n!}{(n-r)!}$$

for $r = 0, 1, 2, \ldots, n$ and n is a positive integer, and $0! = 1$.

Example 3

8

How many ways are there to pick a president, vice-president, and treasurer from 7 people?

A ship has 3 stands and 12 different flags to send signals. How many 3-flag signals can be sent?

What if one or two flags also constitute a signal?

Circular Permutations

The number of permutations of n distinct objects arranged in a circle is $(n-1)!$.

How many ways are there to seat 6 people around table for dinner?

Example 4

10

How many circuits can a traveling saleman make of *n* cities? A reverse route is not considered a unique path.

Nondistinct Permutations

The number of nondistinct permutations of n objects of which n_1 are of the first type, n_2 are of the second type, ..., n_r are of the rth type, is

$$\frac{n!}{n_1! n_2! \cdots n_r!}$$

where $n_1 + n_2 + \cdots + n_r = n$

How many ways are there to line up a pair of identical twins and a set of identical triplets for a photo if identical-looking people are nondistinct?

Example 5

12

How many ways are there to arrange the letters in the word "door"?

How many ways are there to arrange the letters in "puppet"?

How many ways are there to arrange the letters in "wholesome"?

Combination

A set of *r* objects taken from a set of *n* distinct objects without replacement is a **combination**.

List the combinations of 2 elements taken from $\{a, b, c, d\}$.

The number of combinations of *r* objects taken without replacement from *n* distinct objects is

$$\binom{n}{r} = \frac{n!}{(n-r)!r!}.$$

Example 6

14

How many ways are there to pick a **committee** of three people from seven "volunteers"?

How many ways can a five-card hand be dealt from a standard deck of playing cards?

Example 7

15

A ship has 3 stands and 12 flags to send signals. How many signals can be sent if one, two or three flags constitute a signal and the stand selected are relevant?

How many ways can 14 people split into two teams of seven for a game of ultimate frisbee?

1. The well-known **binomial theorem** can be used to expand quantities such as
$$(x + y)^n = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} y^r,$$
where $\binom{n}{r}$ is often referred to as a "binomial coefficient".

2. Several results associated with the binomial coefficients:

    2.1 Symmetry: $\binom{n}{r} = \binom{n}{n-r}$, for $r = 0, 1, \ldots, n$, and n is a positive integer.

    2.2 $\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$

    2.3 $\sum_{r=0}^{k} \binom{m}{r} \binom{n}{k-r} = \binom{m+n}{k}$

3. The binomial coefficient $\binom{n}{r}$ is defined to be 0 when $r < 0$ or $r > n$.

Multinomial

The number of ways of partitioning a set of *n* distinct objects into *k* subsets with n_1 in the first subset, n_2 in the second subset, ..., n_k in the *k*th subset is

$$\binom{n}{n_1, n_2, \ldots, n_k} = \frac{n!}{n_1! n_2! \ldots n_k!}$$

where $n_1 + n_2 + \cdots + n_k = n$.

The Glen family consists of 9 people. How many arrangements are there for them to watch the nightly news seated on four sofas: one that seats three and the other seat two?

Example 8

18

How many ways are there to select 4 billiard balls from a bag containing the 15 balls numbered $1, 2, \ldots, 15$ under the following scenarios?

	Without replacement	With replacement
Ordered sample		
Unordered sample		

THANK YOU!