Department of Mathematics
College of William & Mary

MATH 451/551

Chapter 6. Joint Distribution

6.1 Bivariate Distribution

GuanNan Wang
gwang01@wm.edu

Marginal Distributions

Marginal Distribution

- For the **discrete random variables** X and Y with joint pmf $f(x, y)$, the marginal pmf's for X and Y are
$$f_X(x) = \sum_{y} f(x, y) \quad \text{and} \quad f_Y(y) = \sum_{x} f(x, y)$$
which are defined over the appropriate supports.
- For the **continuous random variables** X and Y with joint pdf $f(x, y)$, the marginal pdf's for X and Y are
$$f_X(x) = \int_{-\infty}^{\infty} f(x, y)dy \quad \text{and} \quad f_Y(y) = \int_{-\infty}^{\infty} f(x, y)dx$$
which are defined over the appropriate supports.

Example 1

Let the discrete random variables X and Y have joint probability mass function

$$f(x,y) = \begin{cases} 0.2 & x=1, y=1 \\ 0.1 & x=1, y=2 \\ 0.3 & x=1, y=3 \\ 0.1 & x=2, y=1 \\ 0.1 & x=2, y=2 \\ 0.2 & x=2, y=3 \end{cases}$$

Discrete.

Find the marginal probability mass functions $f_X(x)$ and $f_Y(y)$.

x \ y	1	2	3	$f_X(x)$
1	0.2	0.1	0.3	0.6
2	0.1	0.1	0.2	0.4
$f_Y(y)$	0.3	0.2	0.5	1

$$f_X(x) = \begin{cases} 0.6 & x=1 \\ 0.4 & x=2 \end{cases}$$

$$f_Y(y) = \begin{cases} 0.3 & y=1 \\ 0.2 & y=2 \\ 0.5 & y=3 \end{cases}$$

①. The marginal pmf of X gives the probability distribution of X when Y is ignored, likewise, the marginal pmf of Y gives the probability distribution of Y. when X is ignored.

②. The marginal pmf/pdf satisfied the same existence conditions as any other pmf/pdf

Example 2

For the continuous random variables X and Y with joint probability density function

Continuous

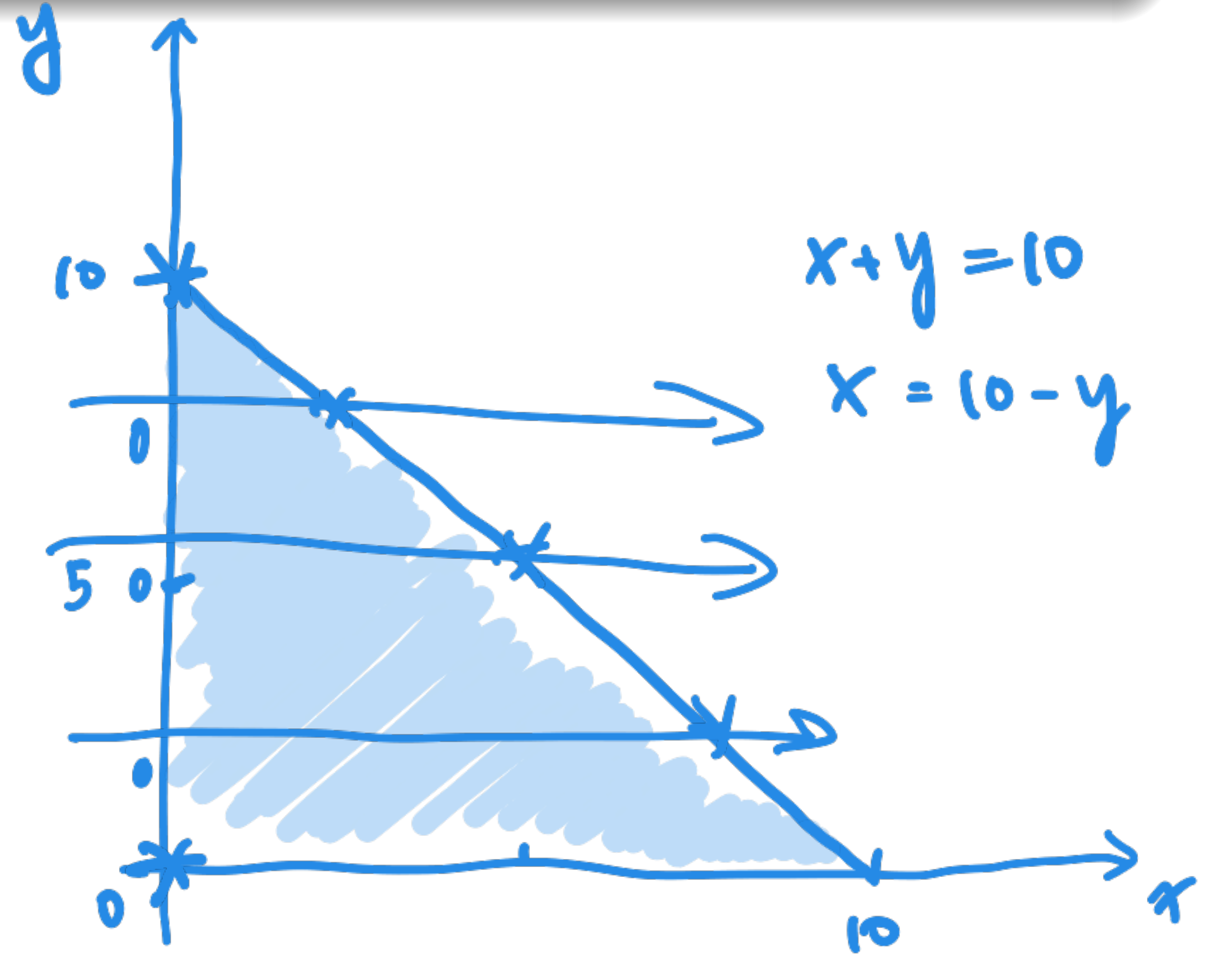
$$f(x,y) = \frac{1}{50}, \quad x > 0,\ y > 0,\ x + y < 10,$$

find the probability density function of the marginal distribution of Y.

$$f_Y(y) = \int_0^{10-y} f(x,y)\,dx$$

$$= \int_0^{10-y} \frac{1}{50}\,dx = \frac{10-y}{50}.$$

$0 < y < 10$



Example 3

Let the continuous random variables X_1 and X_2 have joint probability density function

$$f(x_1, x_2) = x_1 x_2, \quad 0 < x_1 < 1,\ 0 < x_2 < 2.$$

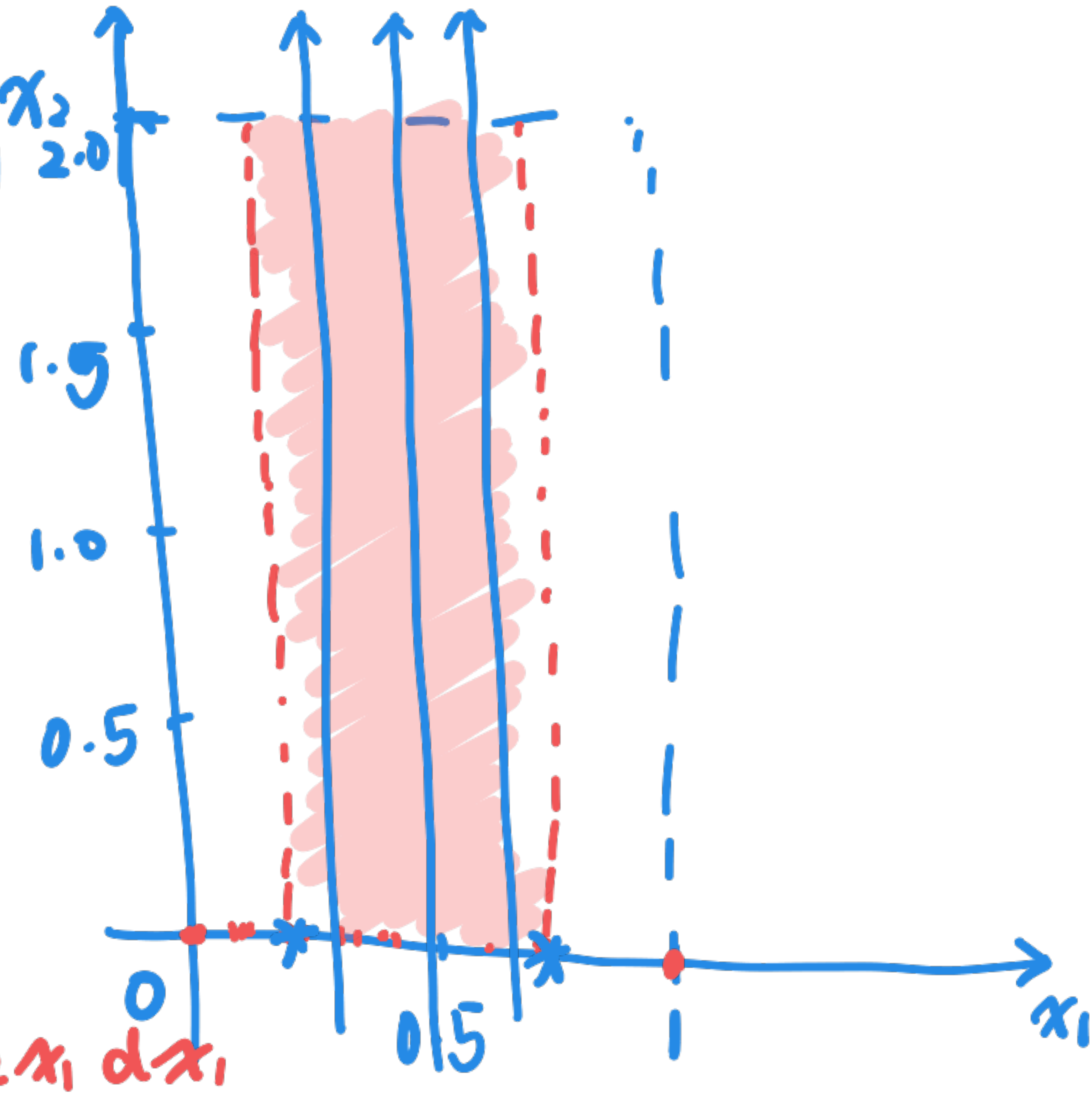
Find $P(0.2 < X_1 < 0.7)$ by the following three techniques:

1. computing the value of the double integral over the appropriate limits
2. finding the marginal probability density function of X_1 and integrating, and
3. finding the marginal cumulative distribution function of X_1 and using the appropriate arguents.

① $P(0.2<X_1<0.7)=\int_{0.2}^{0.7}\int_0^2 f(x_1,x_2)\,dx_2\,dx_1$

$=\int_{0.2}^{0.7}\int_0^2 x_1 x_2\,dx_2\,dx_1 = \int_{0.2}^{0.7}\frac{x_1\cdot\cancel{4}\,2}{\cancel{2}}\,dx_1$

$= x_1^2\Big|_{0.2}^{0.7} = 0.49-0.04=0.45$



② $f_{X_1}(x_1)=\int_0^2 x_1 x_2\,dx_2 = 2x_1,\quad 0<x_1<1$

$P(0.2<X_1<0.7)=\int_{0.2}^{0.7} f_{X_1}(x_1)\,dx_1=\int_{0.2}^{0.7}2x_1\,dx_1$

$=0.45$

③ $F_{X_1}(x_1)=\int_0^{x_1}2x_1\,dx_1 = x_1^2$

$P(0.2<X_1<0.7)=F_{X_1}(0.7)-F_{X_1}(0.2)=0.45$

Thank you!