

MATH 451/551

Chapter 4. Common Discrete Distributions

4.3 Geometric Distribution

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Geometric Distribution

i.i.d. Bernoulli trials
 $X = \#$ failure before
 1st success.



Geometric Distribution

- ▶ A geometric random variable X is the number of failures before the first success in repeated independent and identically distributed Bernoulli trials.

- ▶ **Support:** $\mathcal{A} = \{0, 1, 2, \dots\}$.

- ▶ **Probability mass function (PMF):**

1st trial F 2nd trial F 3rd ... xth S
 $(1-p) * (1-p) * (1-p) \dots (1-p) * p$

Bernoulli trial outcomes	x value	$f(x) = P(X = x)$
<u>S</u>	<u>0</u>	<u>p</u>
<u>FS</u>	<u>1</u>	<u>$p(1-p)$</u>
<u>FFS</u>	<u>2</u>	<u>$p(1-p)^2$</u>
<u>FFFS</u>	<u>3</u>	<u>$p(1-p)^3$</u>
...	...	...

- ▶ A discrete random variable X with PMF $f(x) = p(1-p)^x$, $x = 0, 1, 2, \dots$ for $0 < p < 1$ is a *Geometric(p)* random variable.

$X \sim \text{Geometric}(p)$
 $X \sim \text{Geo}(p)$

- ▶ **Shorthand:** $X \sim \text{Geometric}(p)$ or $X \sim \text{Geo}(p)$.

CDF

$$\underbrace{p \times 1 \quad p(1-p) \quad p(1-p)^2 \quad \dots \quad p(1-p)^x}_{(1-p)^0 \quad (1-p)^1 \quad (1-p)^2 \quad \dots \quad (1-p)^x}$$



$$F_X(x) = P(X \leq x)$$

$$= \sum_{w=0}^x f(w) = \sum_{w=0}^x p(1-p)^w = p \sum_{w=0}^x (1-p)^w$$

$$= p \frac{1 \times (1 - (1-p)^{x+1})}{1 - (1-p)} = \cancel{p} \frac{1 - (1-p)^{x+1}}{\cancel{p}}$$

$$= 1 - \underline{(1-p)^{x+1}}$$

$$x \in \mathcal{A}$$

Thank You



THANK YOU!