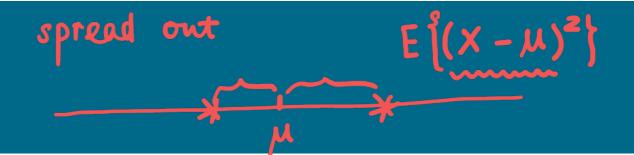
MATH 451/551

Chapter 3. Random Variables

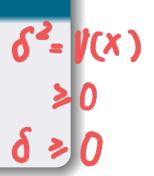
Moments





Variance

For a random variable X with population mean μ , the **population** variance of X is $\sigma^2 = V(X) = E\{(X - \mu)^2\}$ when the expected values exist.



- ► The units on the population variance are square of the units of the random variable *X*.
- The positive square root of the population variance is the **population standard deviation** σ . One reason for the popularity of σ is that its units are the same units as the random variable X.
- If the discrete random variable X has a support \mathcal{A} that contains only one x-value, then $\sigma^2 = 0$. This distribution is often known as a **degenerate** distribution.
- ightharpoonup Some authors use V(X) for the population variance.
- The population variance is not the only measure of dispersion for a random variable X.
 - ▶ the population range $R = \sup(A) \inf(A)$,
 - the population mean absolute deviation $E(|X \mu|)$.

Moments



Variance

For the random variable X with population mean μ and population variance σ^2 , $V(X) = E(X^2) - \mu^2$, when is known as the **shortcut formula** for computing the population variance.

$$V(X) = E\{(X - \mu)^{2}\}$$

$$= E\{(X^{2} - 2\mu X + \mu^{2}) = E(X^{2}) - E(X^{2}\mu X) + E(X^{2}\mu^{2})$$

$$= E(X^{2}) - 2\mu E(X) + \mu^{2}$$

$$= E(X^{2}) - 2\mu \mu + \mu^{2}$$

$$= E(X^{2}) - \mu^{2}$$
and population moment.



Example 11

Using the defining and computational formulas, find the population variance of the number of spots showing when rolling a fair die.



Example 12

Calculate the population variance of the random variable X with probability density function $\mu = \int_{a}^{\infty} \pi f(x) dx = \int_{a}^{\infty}$

$$f(x) = \frac{x}{2}, \quad 0 < x < 2$$

using the defining formula and the computation formula.

$$\delta^{2} = E\left[\frac{(X-M)^{2}}{(X-M)^{2}}\right] = \int_{M} (x^{2} - \frac{4}{3})^{2} \frac{x}{2} dx \qquad E\left(\frac{x^{2}}{M}\right) = \int_{M} x^{2} \frac{x}{2} dx = \int_{0}^{2} \frac{x^{3}}{2} dx$$

$$= \int_{0}^{2} \left(\frac{x^{3}}{M} - \frac{4x^{2}}{3} + \frac{16x^{2}}{189}\right) dx$$

$$= \frac{x^{4}}{8} \Big|_{0}^{2} - \frac{4x^{3}}{9} \Big|_{0}^{2} + \frac{4x^{2}}{9} \Big|_{0}^{2}$$

$$= \frac{16}{8} - \frac{32}{9} + \frac{16}{9}$$

$$= \frac{16}{8} - \frac{16}{9} = \frac{2}{9}$$

Properties of Variance



Property 7

For the random variable X with population mean μ and population variance σ^2 ,

for real constants a and b.

$$\Gamma\left(\frac{ax+b}{x}\right) = E \left[\frac{(ax+b)-E(ax+b)}{2x}\right] = E \left[\frac{ax+b-E(ax)+E(b)}{2x}\right]$$

$$= E \left[\frac{ax+b-aE(x)}{2x}\right] = a^2 E\left[\frac{ax+b-E(x)}{2x}\right]$$

$$= a^2 \Gamma(x)$$

$$0 = 0$$

$$U(\alpha X) = 0^2 V(X)$$



Example 13

Random variable X has the probability density function

$$f(x) = \begin{cases} \frac{3}{2}x^2 + x, & 0 \le x \le 1 \\ 0, & \text{o.w.} \end{cases}$$

Find V(X).



Example 14

Consider the random variable X with cumulative distribution function

$$F(x) = \begin{cases} 0, & x < -1 \\ \frac{x}{5} + \frac{3}{10}, & -1 \le x < 0 \\ \frac{2}{5}, & 0 \le x < 1 \\ \frac{x}{5} + \frac{2}{5}, & 1 \le x < 3 \\ 1, & x \ge 3 \end{cases}.$$

Find E(X) and V(X).



Example 15

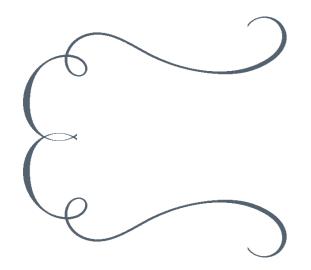
Let Y be a random variable of the continuous type with PDF f(y), which is positive provided 0 < y < b < 1, and is equal too zero elsewhere. Show that

$$E(Y) = \int_0^b \{1 - F(y)\} dy,$$

where F(y) is the cumulative distributioon function of Y.

Thank You





THANK YOU!

