Department of Mathematics
College of William & Mary

MATH 451/551

Chapter 3. Random Variables

3.4 Expected Values

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X is continuous R.V. $\mathcal{A}$.
$f_X(x)$ then $\mu = E(X) = \int_{\mathcal{A}} x f(x)\,dx$.

Example 5

Find the population mean of the random variable X with probability density function

$$f(x) = \frac{x}{2}, \qquad 0 < x < 2.$$ $\Rightarrow$ X is continuous.

$$\mu = E(X) = \int_{\mathcal{A}} x f(x)\,dx = \int_0^2 x\,\frac{x}{2}\,dx = \int_0^2 \frac{x^2}{2}\,dx = \left.\frac{x^3}{6}\right|_0^2$$

$$= \frac{8}{6} - 0 = \frac{4}{3}$$

Example 6

For the random variable X with probability density function

$$f(x) = \begin{cases} 1 - x & 0 < x < 1 \\ 1/4 & 5 < x < 7 \end{cases}$$

calculate $\mu = E(X)$.

$$f_X(x) = \begin{cases} 1-x & 0<x<1 \\ 1/4 & 5<x<7 \\ 0 & \text{o.w. (otherwise)}. \end{cases}$$

$$\mu = E(X) = \int_{\mathbb{R}} x f_X(x)\,dx = \int_0^1 x(1-x)\,dx + \int_5^7 x \frac{1}{4}\,dx$$

$$= \left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^1 + \frac{x^2}{8}\Big|_5^7 = \frac{1}{2} - \frac{1}{3} + \frac{49-25}{8} = \frac{1}{6} + \frac{24}{8} = \frac{19}{6}$$

Example 7

Find $E(X)$ for the random variable X with proobability density function

$$f(x) = \frac{1}{x^2}, \quad x > 1. \Rightarrow X \text{ is continuous.}$$

$$\mu = E(X) = \int_{S} x f(x)\,dx = \int_1^{\infty} x \frac{1}{x^2}\,dx = \int_1^{\infty} \frac{1}{x}\,dx.$$

does not exist.

Example 8

Find $E(X)$ for the random variable X with probability density function

$$f(x) = \frac{1}{\pi(1+x^2)}, \quad -\infty < x < \infty.$$

Cauchy distribution

$\Rightarrow X$ is continuous.

$$\mu = E(X) = \int_{\mathbb{R}} x f(x)\,dx = \int_{-\infty}^{\infty} \frac{x}{\pi(1+x^2)}\,dx = \frac{1}{2\pi} \ln(1+x^2)\Big|_{-\infty}^{\infty}$$

does NOT exist.

$E(X)$ doesnot exist for Cauchy distribution.

Thank You

THANK YOU!