

MATH 451/551

Chapter 3. Random Variables

3.3 PDF & CDF

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uniform distribution

$$U\left(\underset{\substack{\uparrow \\ \text{lb of } \mathcal{A}}}{a}, \underset{\substack{\uparrow \\ \text{ub of } \mathcal{A}}}{b}\right)$$

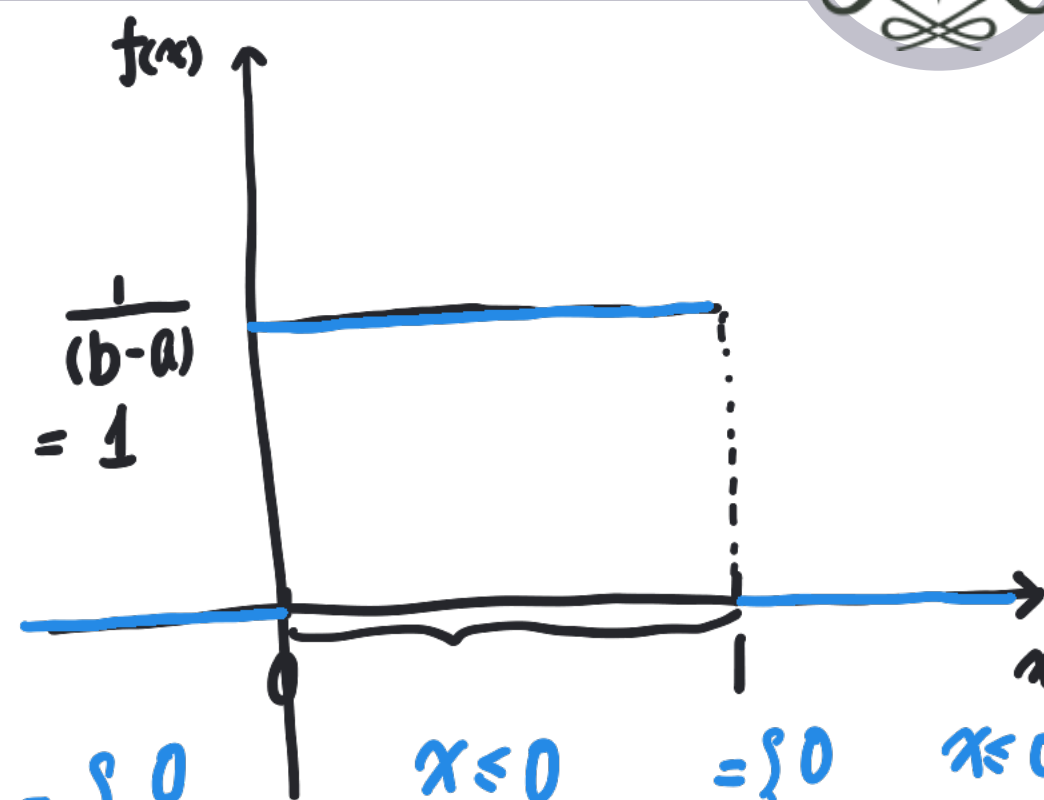
$$X \sim U(0,1)$$

follows

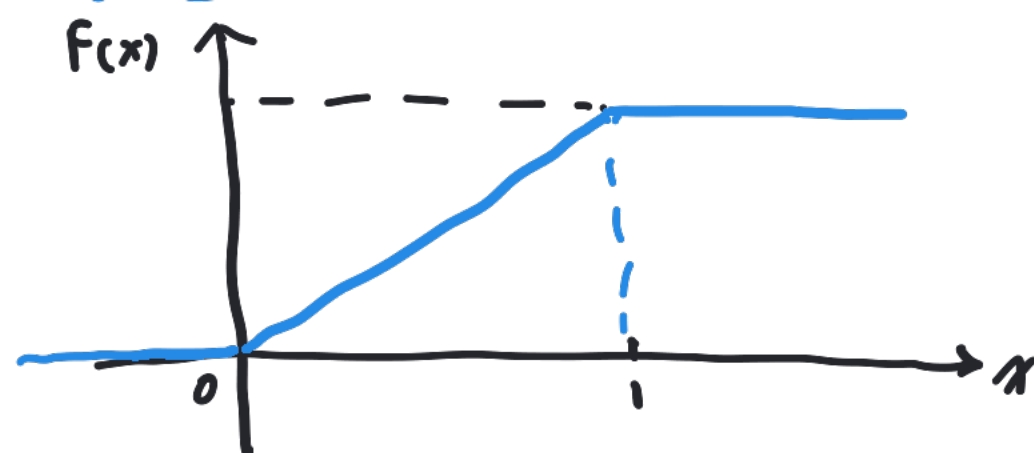
$$f_X(x) = \frac{1}{b-a} = \frac{1}{1-0} = 1, \quad x \in \underbrace{(0,1)}_{\mathcal{A}}$$

$$f_X(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{o.w. (otherwise)} \end{cases}$$

$$F_X(x) = P(X \leq x) = \begin{cases} \int_{-\infty}^0 0 dx & , x \leq 0 \\ \int_{-\infty}^0 0 dx + \int_0^x 1 dx, & 0 < x < 1 \\ \int_{-\infty}^0 0 dx + \int_0^1 1 dx + \int_1^x 0 dx & , x \geq 1 \end{cases}$$



$$F_X(x) = \begin{cases} 0 & x \leq 0 \\ 0 + x & 0 < x < 1 \\ 0 + 1 + 0 & x \geq 1 \end{cases} = \begin{cases} 0 & x \leq 0 \\ x & 0 < x < 1 \\ 1 & x \geq 1 \end{cases}$$





flip a balanced coin. $X = \# \text{ head.}$

$$X = \begin{cases} 1 \\ 0 \end{cases} \Rightarrow \mathcal{A} = \{0, 1\}$$

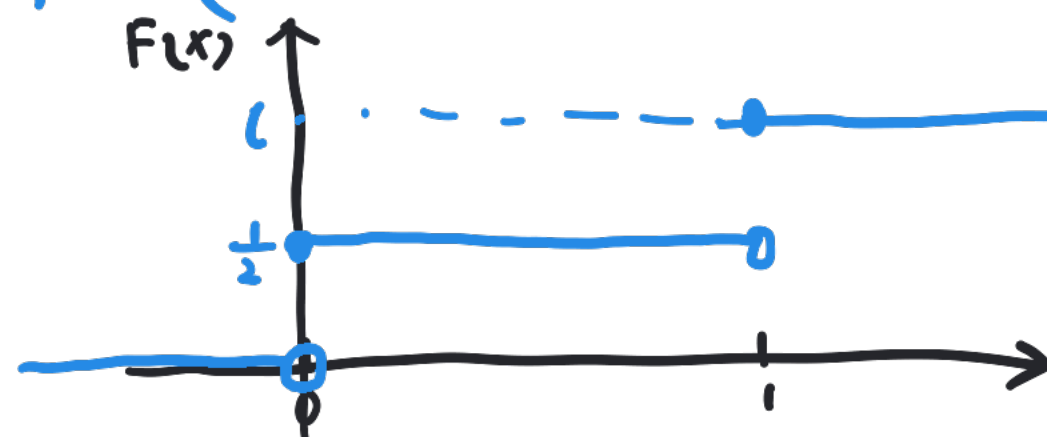
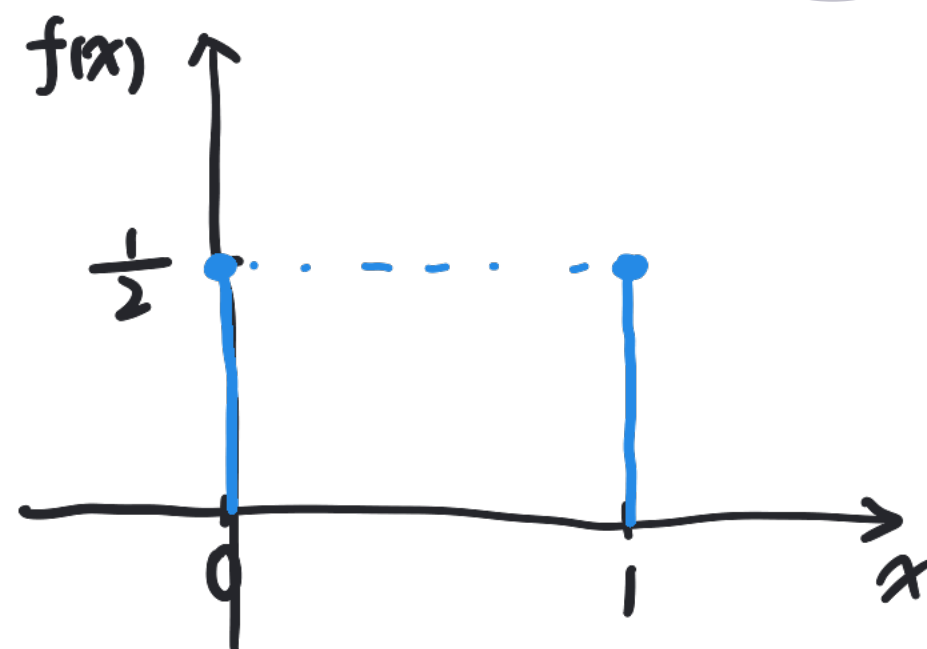
$$f_X(x) = \frac{1}{2} \quad x=0, 1$$

$$f_X(x) = \begin{cases} \frac{1}{2} & x=0, 1 \\ 0 & \text{o.w.} \end{cases}$$

$$F_X(x) = P(X \leq x) = \sum_{w \leq x} 0$$

$$\begin{cases} \sum_{w < 0} 0 + \sum_{w=0} \frac{1}{2} + \sum_{0 < w < 1} 0 & 0 \leq x < 1 \\ \sum_{w < 0} 0 + \sum_{w=0} \frac{1}{2} + \sum_{0 < w < 1} 0 + \sum_{w=1} \frac{1}{2} + \sum_{w>1} 0 & x \geq 1 \end{cases}$$

$$F_X(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2} & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$



Thank You



THANK YOU!