

MATH 451/551

Chapter 3. Random Variables

3.3 Cumulative Distribution Function

$$F_X(x) = P(X \leq x)$$

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Example 5



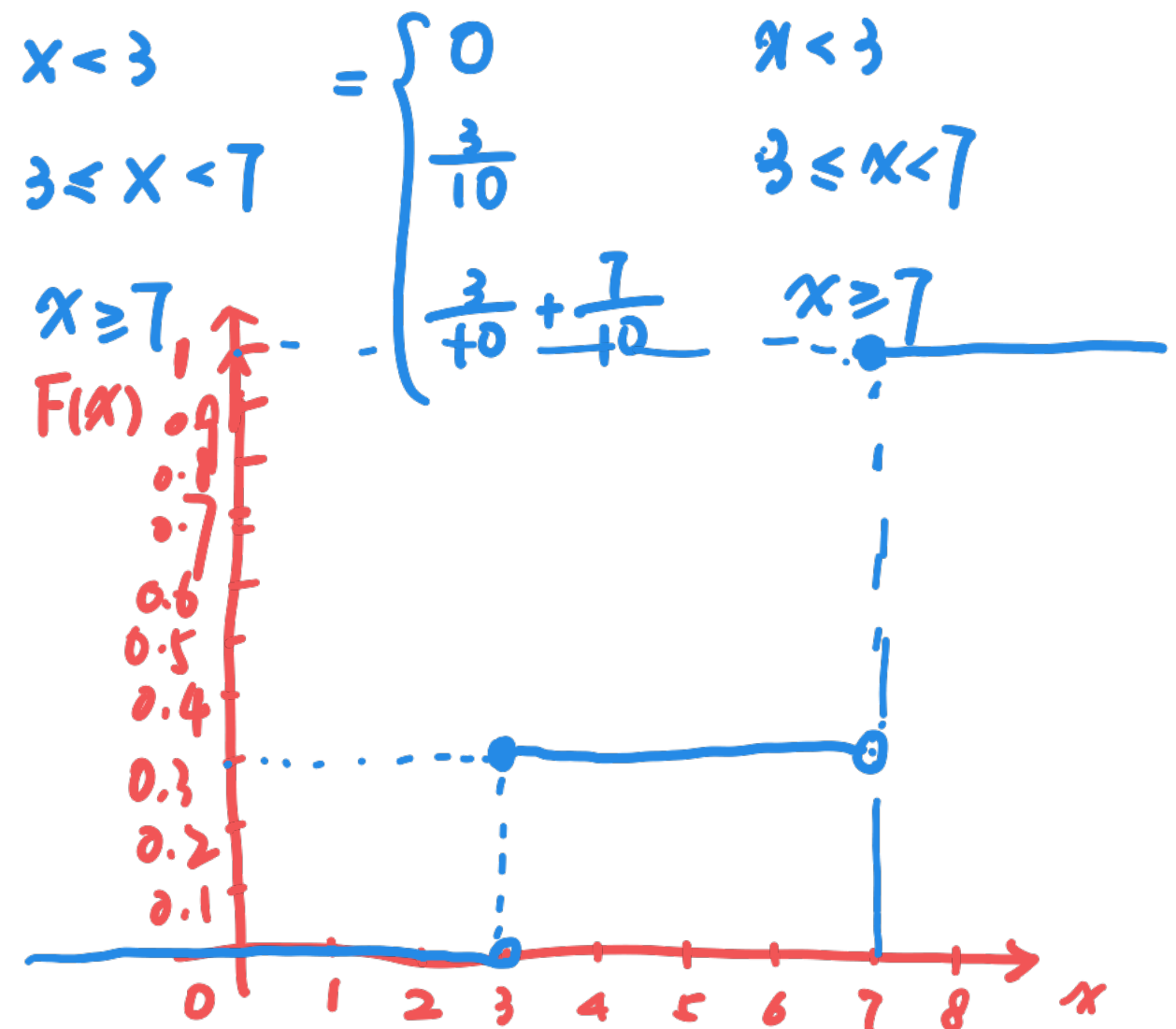
Find the cumulative distribution function for the random variable X with probability mass function

$$f(x) = x/10, \quad x = 3, 7.$$

$\mathcal{X} = \{3, 7\} \Rightarrow X$ is discrete.

$$F_X(x) = \sum_{w \leq x} f(w) = \begin{cases} 0 & x < 3 \\ \sum_{w \leq 3} f(w) & 3 \leq x < 7 \\ \sum_{w \leq 7} f(w) & x \geq 7 \end{cases}$$

$$= \begin{cases} 0 & x < 3 \\ 0.3 & 3 \leq x < 7 \\ 1 & x \geq 7 \end{cases}$$



Example 6

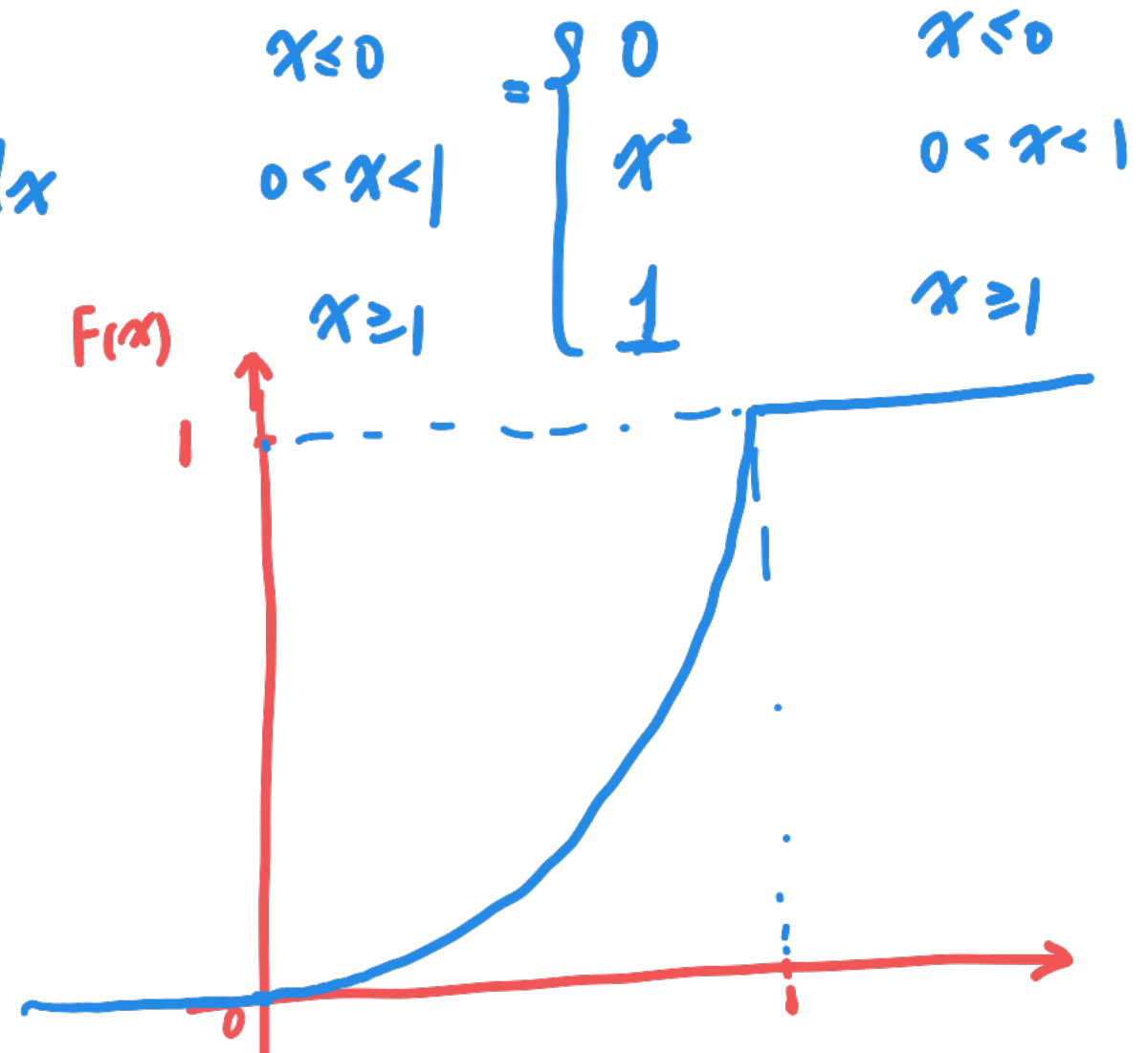


Find the cumulative distribution function for the random variable X with probability density function

$$f(x) = 2x, \quad 0 < x < 1.$$

$\mathcal{A} = \{0 < x < 1\} \Rightarrow X$ is continuous!

$$F_X(x) = \begin{cases} 0 & x \leq 0 \\ \int_{-\infty}^x f(w) dw & 0 < x < 1 \\ 1 & x \geq 1 \end{cases} = \begin{cases} 0 & x \leq 0 \\ \int_0^x 2x dx & 0 < x < 1 \\ 1 & x \geq 1 \end{cases}$$





Find the cumulative distribution function for the random variable X with probability density function

$$f_X(x) = \begin{cases} 1-x & 0 < x < 1 \\ 1/4 & 5 < x < 7 \end{cases}.$$

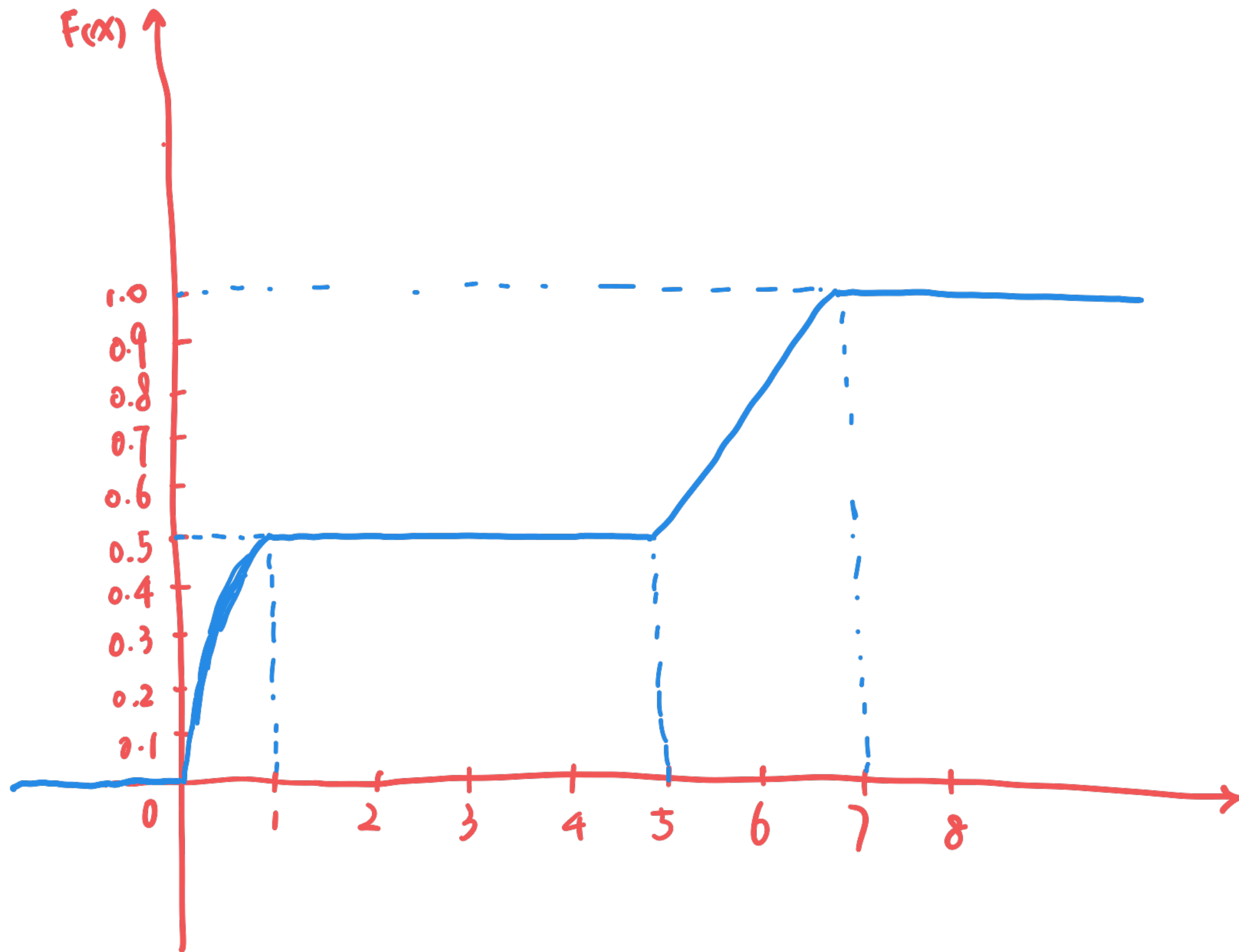
$x < 1, 5 < x < 7 \Rightarrow X$ is continuous.

$$\begin{aligned} & \int_{-\infty}^x f(w) dw \\ & \int_{-\infty}^1 f(w) dw \\ & \int_{-\infty}^x f(w) dw \\ & 1 \end{aligned}$$

$$\begin{aligned} & x \leq 0 \\ & 0 < x < 1 \\ & 1 \leq x \leq 5 \\ & 5 < x < 7 \\ & x \geq 7 \end{aligned} \quad \begin{cases} \int_0^x (1-w) dw \\ \int_0^1 (1-w) dw \\ \int_0^1 (1-w) dw + \int_5^x \frac{1}{4} dw \\ 1 \end{cases}$$

$$\begin{aligned} & x \leq 0 \\ & 0 < x < 1 \\ & 1 \leq x \leq 5 \\ & 5 < x < 7 \\ & x \geq 7 \end{aligned}$$

$$\begin{aligned} & -\int_0^x (1-w) dw = -\left[\frac{w^2}{2} - w \right]_0^x = -\left(\frac{x^2}{2} - x \right) = -\frac{x^2}{2} + x \\ & -\int_0^1 (1-w) dw = -\left[\frac{w^2}{2} - w \right]_0^1 = -\left(\frac{1}{2} - 1 \right) = \frac{1}{2} \\ & -\int_0^1 (1-w) dw - \int_5^x \frac{1}{4} dw = \frac{1}{2} - \frac{x-5}{4} = \frac{x-3}{4} \\ & 1 \end{aligned}$$



Thank You



THANK YOU!