Department of Mathematics
College of William & Mary

MATH 451/551

Chapter 3. Random Variables

3.1 Discrete Random Variables

R.V. $\begin{cases} \text{discrete R.V} \\ \text{continuous R.V.} \end{cases}$

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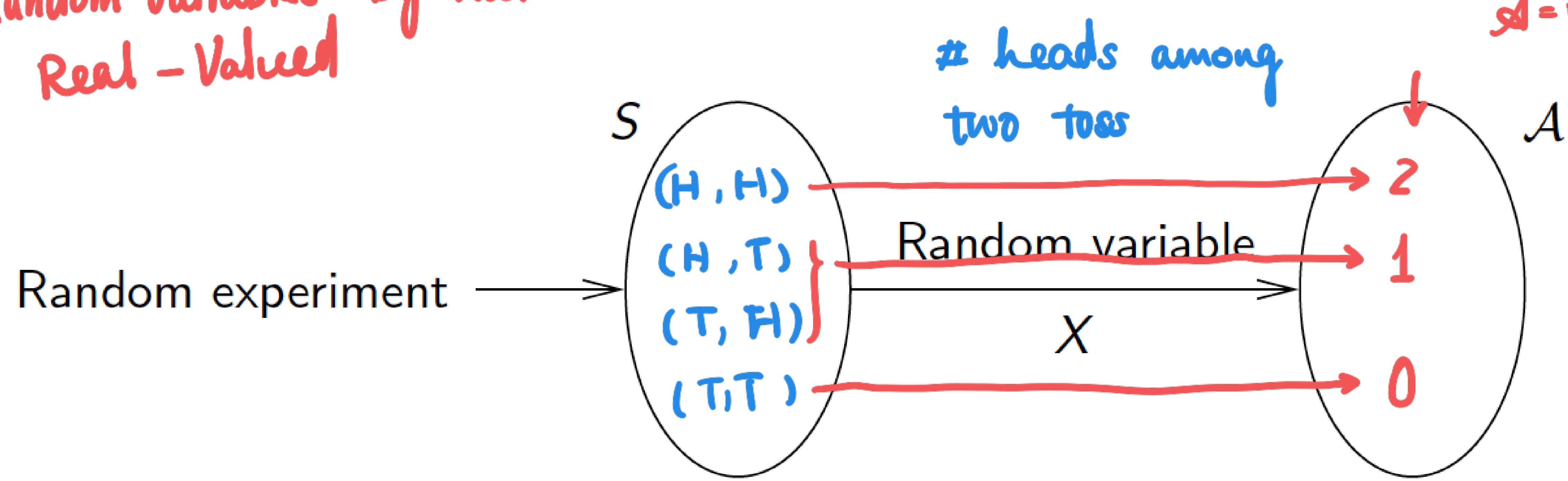
Random Variables

Given a random experiment with an associated sample space S, a **random variable** is a function X that assigns to each element $s \in S$ one and only one real number $X(s) = x$. The **support** of X is the set of real numbers $\mathcal{A} = \{x \mid x = X(s),\ s \in S\}$.

A fair coin is tossed twice. Find the probability that both tosses come up heads.

Random variables by nature Real-Valued

$\mathcal{A} = \{0, 1, 2\}$



x	0	1	2
$P(X=x)$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

probability distribution.

x – value

$P(X = x)$

$\uparrow$ R.V. $\quad \uparrow$ value

$P(X=0) = P(x) = P(0) = \frac{1}{4}$

$P(X=1) = P(1) = \frac{2}{4}$

$P(X=2) = P(2) = \frac{1}{4}$

Discrete Random Variable

Discrete Random Variable

Let X denote a random variable with one-dimensional support $\mathcal{A}$, then a random variable X is discrete if the support $\mathcal{A}$ is countable, that is

- X is a discrete random variable if $\mathcal{A}$ is a finite set, or
- X is a discrete random variable if $\mathcal{A}$ is a denumerable set.

Probability Mass Function (PMF) $f(x) = P(X = x)$

- The probability that X takes on the value x, $P(X = x)$, is defined as the sum of the probabilities of all sample points in S that are assigned the value x. Sometimes, we denote $P(X = x) = p(x)$.
- The probability distribution for a discrete random variable (r.v.) X can represented by a formula, a table, or a graph which provides $P(X = x) = p(x)$ for all x, such that
  - $0 \leq p(x) \leq 1$, $\forall x$
  - $\sum_x p(x) = 1$, where Σ_x is the sum over all possible values, x, of the r.v. x

Example 1

Toss a fair coin twice times. Find the probability distribution for the number of heads.

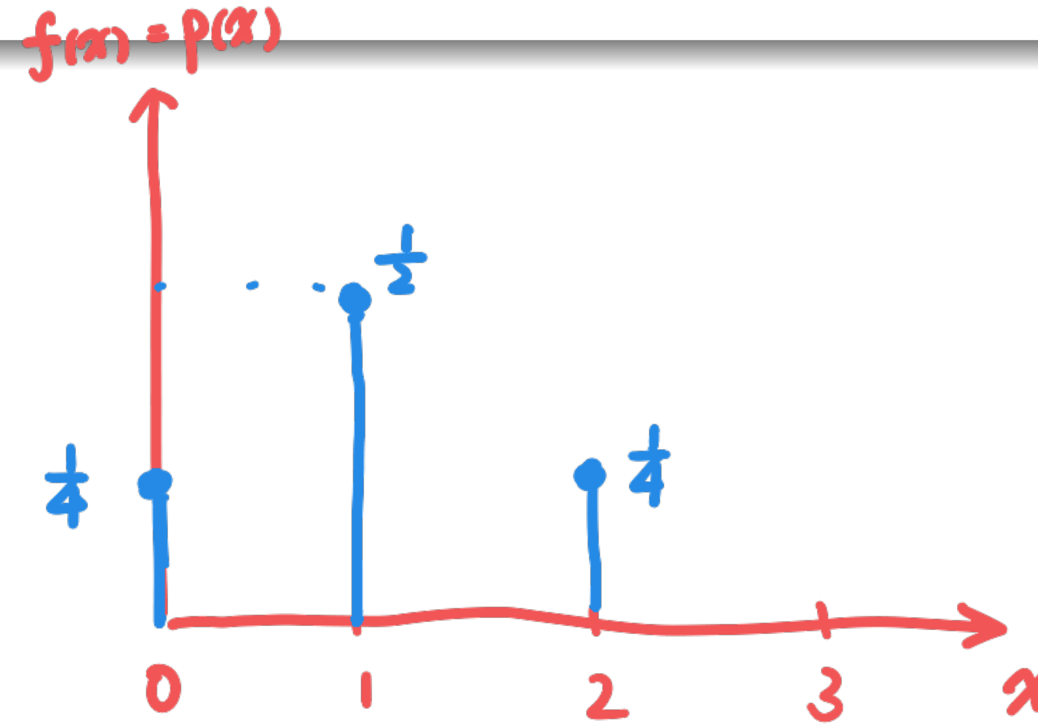
X = # heads among two tosses

x	0	1	2
$P(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$\mathcal{A}$

check $p(x) \in [0,1]$ ✓

$$\sum_{x \in \mathcal{A}} p(x) = 1$$



$$f(x) = P(X=x) = \begin{cases} \frac{1}{4}, & x=0 \\ \frac{1}{2}, & x=1 \\ \frac{1}{4}, & x=2 \end{cases} = \underbrace{\binom{2}{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{2-x}, \quad x=0,1,2}_{\text{binomial R.V.}}$$

Example

Example 2

A 5-card hand is dealt from a well-shuffled from a 52-card deck. Let the random variable X be the number of jacks in the hand. Find the support of X and the probability mass function of X.

Example 3

A box contains 6 green balls and 10 red balls. If we draw 2 balls without replacemennt from the box, let $X = \#$ of green balls drawn, find the probability mass function of X.

Example

Example 4

A spinner yields three equally-likely outcomes: 1, 2, 3. If the random variable X denotes the product of the outcomes of two spins, find the probability mass function $f(x)$, $P(X = 6)$, and $P(X \leq 6)$.

Example 5

Flip a fair coin repeatedly until a head appears. Let X be the number of flips required. Find the probability mass function of X and find $P(X \geq 4)$.

THANK YOU!