Department of Mathematics
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MATH 451/551

Chapter 2. Introduction

2.3 Computing Probabilities

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Example 1

Three men and two women sit in a row of chairs in a random order. Let the event A be that men and women alternate (that is, MWMWM). Find $P(A)$.

$$P(A) = \frac{N(A)}{N(S)} = \frac{12}{5!} = \frac{\cancel{12}}{5\times\cancel{4\times3}\times2\times1} = \frac{1}{10}$$

$N(S)$:

	1st	2nd	3rd	4th	5th	
$N(S)$:	$5 \times$	$4 \times$	$3 \times$	$2 \times$	1	$= 5!$
	M	W	M	W	M	
	1st	2nd	3rd	4th	5th	
$N(A)$:	$3 \times$	$2 \times$	$2 \times$	$1 \times$	1	$= 12$

Example 2

Karen collects n hats and returns them at random. Let the event A be the proper return of the hats to their owners. Find $P(A)$.

$$P(A) = \frac{N(A)}{N(S)} = \frac{1}{n!}$$

1st 2nd ... nth

$N(S)$: $n \times (n-1) \times \cdots \times 1 = n!$

$N(A)$: $1 \times 1 \times \cdots \times 1 = 1$

Example 3

Roll a pair of fair dice 24 times. Let the event A be rolling double aces (that is, double ones) at least once. Find $P(A)$.

$$P(A) = \frac{N(A)}{N(S)}$$

A^c = not rolling double Aces

$$= 1 - P(A^c) = 1 - \frac{N(A^c)}{N(S)} = 1 - \left(\frac{35}{36}\right)^{24}$$

1st 2nd ... 24th

$N(S)$: 6×6

$$36 \times 36 \times \cdots \times 36 = 36^{24}$$

$$N(A^c) \quad 35 \times 35 \times \cdots \times 35 = 35^{24}$$

Example 4

A five-card poker hand is dealt from a well-shuffled deck. Let the event A be that there are exactly 2 kings in the hand. Find $P(A)$.

$$P(A) = \frac{N(A)}{N(S)} = \frac{\binom{4}{2}\binom{48}{3}}{\binom{52}{5}} = \frac{103776}{2598960} = 0.0399$$

$$N(S) = \binom{52}{5}$$

kings other cards

$$N(A) = \binom{4}{2} \times \binom{48}{3}$$

Example 5

A five-card poker hand us dealt from a well-shuffled~~d~~ deck. Let the event A be dealing a full house. Find $P(A)$.

full house. $\underbrace{\heartsuit 7 \quad \clubsuit 7 \quad \diamondsuit 7}$ $\quad \underbrace{\clubsuit J \quad \clubsuit J}$

$$P(A) = \frac{N(A)}{N(S)} = \frac{13\binom{4}{3}12\binom{4}{2}}{\binom{52}{5}} = 0.00144$$

kind. 3 cards kind pair.

$$N(A) = \underline{13 \times \binom{4}{3} \times 12 \times \binom{4}{2}}$$

$$\underbrace{2, 3, \cdots, 10, J, Q, K, A}_{13 \text{ kinds}}$$

Example 6

A five-card poker hand is dealt from a well-shuffled deck. Let the event A be dealing two pair. Find $P(A)$.

$$P(A) = \frac{N(A)}{\underset{\text{kind}}{N(S)}} = \frac{\binom{13}{2}\binom{4}{2}\binom{4}{2}44}{\binom{52}{5}} \approx 0.0475$$

$\heartsuit 7 \quad \clubsuit 7 \quad \spadesuit J \quad \diamondsuit J \quad \boxed{\clubsuit A}$

(-4) (-4) 44

$$N(A) = \binom{13}{2}\binom{4}{2}\binom{4}{2}\overbrace{\binom{11}{1}\binom{4}{1}}^{\text{other card}}$$

$$= \binom{13}{2}\binom{4}{2}\binom{4}{2}44$$

Computing Probabilities

Example 7

A five-card poker hand is dealt from a well-shuffled deck. Let the event A be dealing a straight. Find $P(A)$.

♡8 ♡9 ♤10 ◇J ♧Q

$$P(A) = \frac{N(A)}{N(S)} = \frac{10 \times (4^5 - 4)}{\binom{52}{5}} \approx 0.0039$$

all ♤
all ♡
all ♧
all ◇

seq × suits

$N(A) = 10 \times 4 \times 4 \times 4 \times 4 \times 4 - 4$

$= 10 \times (4^5 - 4)$

2	3	4	5	6	1
3	4	5	6	7	2
4	5	6	7	8	3
5	6	7	8	9	4
6	7	8	9	10	5
7	8	9	10	J	6
8	9	10	J	Q	7
9	10	J	Q	K	8
10	J	Q	K	A	9
A	2	3	4	5	10

Computing Probabilities

Example 8

A five-card poker hand is dealt from a well-shuffled deack. Let the event A be dealing two pair. Find $P(A)$.

Example 9

A dozen eggs contains 3 defectives. If a sample of 5 is selected from the dozen at random,

1. find the probability that the sample contains exactly 2 defectives,
2. find the probability that the sample contains 2 or fewer defectives.

1 0

let A = the sample contains exactly 2 def.
B = the sample contains 2 or fewer def

A_i = sample contains exactly i def
$i = 2, 1, 0$

$$P(A) = \frac{N(A)}{N(S)} = \frac{\overset{\text{def}}{\binom{3}{2}} \times \overset{\text{good}}{\binom{12-3}{5-2}}}{\binom{12}{5}} = \frac{252}{792} \approx 0.3182$$

$$P(B) = P(A_0 \cup A_1 \cup A_2) = P(A_0) + P(A_1) + P(A_2)$$

$$= \frac{\overset{\text{good}}{\binom{9}{5}}}{\binom{12}{5}} + \frac{\overset{\text{def}}{\binom{3}{1}} \times \overset{\text{good}}{\binom{9}{4}}}{\binom{12}{5}} + \frac{\binom{3}{2} \times \binom{9}{3}}{\binom{12}{5}} \approx 0.9545$$

Computing Probabilities

Example 10

A bag contains 15 billiard balls numbered 1 through 15. Five balls are randomly drawn from the bag without replacement. Let the event A be exactly two odd-numbered balls are drawn from the bag and they occur on off-numbered draws. Find $P(A)$.

THANK YOU!