Department of Mathematics
College of William & Mary

MATH 451/551

Chapter 2. Introduction

2.1 Random Experiments, Sample Spaces, and Events

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Random Experiment

Random Experiment

A *random experiment* is one in which the outcome is subject to chance. Every possible outcome can typically be described prior to the execution of the random experiment.

Associated with a random experiment is the set of all possible outcomes to that experiment. For example,

- when a quarterback throws a pass in football, there are three possible outcomes: a complete pass, an incomplete pass, and an interception;
- when a gambler bets on “red" in roulette, there are two possible outcomes: winning and losing;
- when a backgrammon player rolls a pair of dice, there are 11 different sums that are possible: 2, 3, . . ., 12.

Sample Space

Sample Space

The set of all possible ooutcomes too oa random experiment is called the **sample space** and is denoted by S.

Random Experiment	Sample Space
Roll a die and observe up face	$S=\{1,2,3,4,5,6\}$
Roll two dice and observe the product of the up faces	
Roll a red die and a green die and observe the difference beween the red up face and the green up face	
Roll three dice and observe the sum of the up faces	$S=\{3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18\}$
Toss a coin twice and observe the sequence of H's and T's	$S=\{(H,H),(H,T),(T,H),(T,T)\}$
Toss a coin twice and observe the number of H's	$S=\{2,1,0\}$

D_1^R D_2^G

D_1	D_2	
1	1	0
1	2	-1
1	3	-2
1	4	-3
1	5	-4
1	6	-5
2	1	1
2	2	0
2	3	-1
2	4	-2
2	5	-3
2	6	-4
3	1	2
3	2	1
3	3	0
3	4	-1
3	5	-2
3	6	-3

$S = \{1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 30, 36\}$

$S = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

D_1	D_2	
4	1	3
4	2	2
4	3	1
4	4	0
4	5	-1
4	6	-2
5	1	4
5	2	3
5	3	2
5	4	1
5	5	0
5	6	-1
6	1	5
6	2	4
6	3	3
6	4	2
6	5	1
6	6	0

4	3	1
8	2	2
12	1	3
16	0	4
20	-1	5
24	-2	6
5	4	1
10	3	2
15	2	3
20	1	4
25	0	5
30	-1	6
6	5	1
12	4	2
18	3	3
24	2	4
30	1	5
36	0	6

Remarks

- The sample space: in the above examples have one common attribute: they all correspond to sets that are known as **finite sets**. Each samples space has a finite number of elements.
- A set that is not finite is known as an **infinite set**. Furthermore, a set is denumerable if its elements can be placed in a one-to-one correspondence with the natural numbers.

- A set is **countable** if it is either finite or denumerable.
- A set is **uncountable** if it is not countable.

Event

Event

An **event** is any collection of possible outcomes of an experiment, that is, any subset of S (includes S itself). Let the **event** A be a subset of the sample space S. If an outcome to a random experiment is in A then event A has occurred.

- A random experiment has multiple events.
- The event with the smallest number of outcomes is the empty set Φ, which never occurs.

Set Theory	Probability
universal set	sample space
elements	outcomes of a random experiment
subsets	events

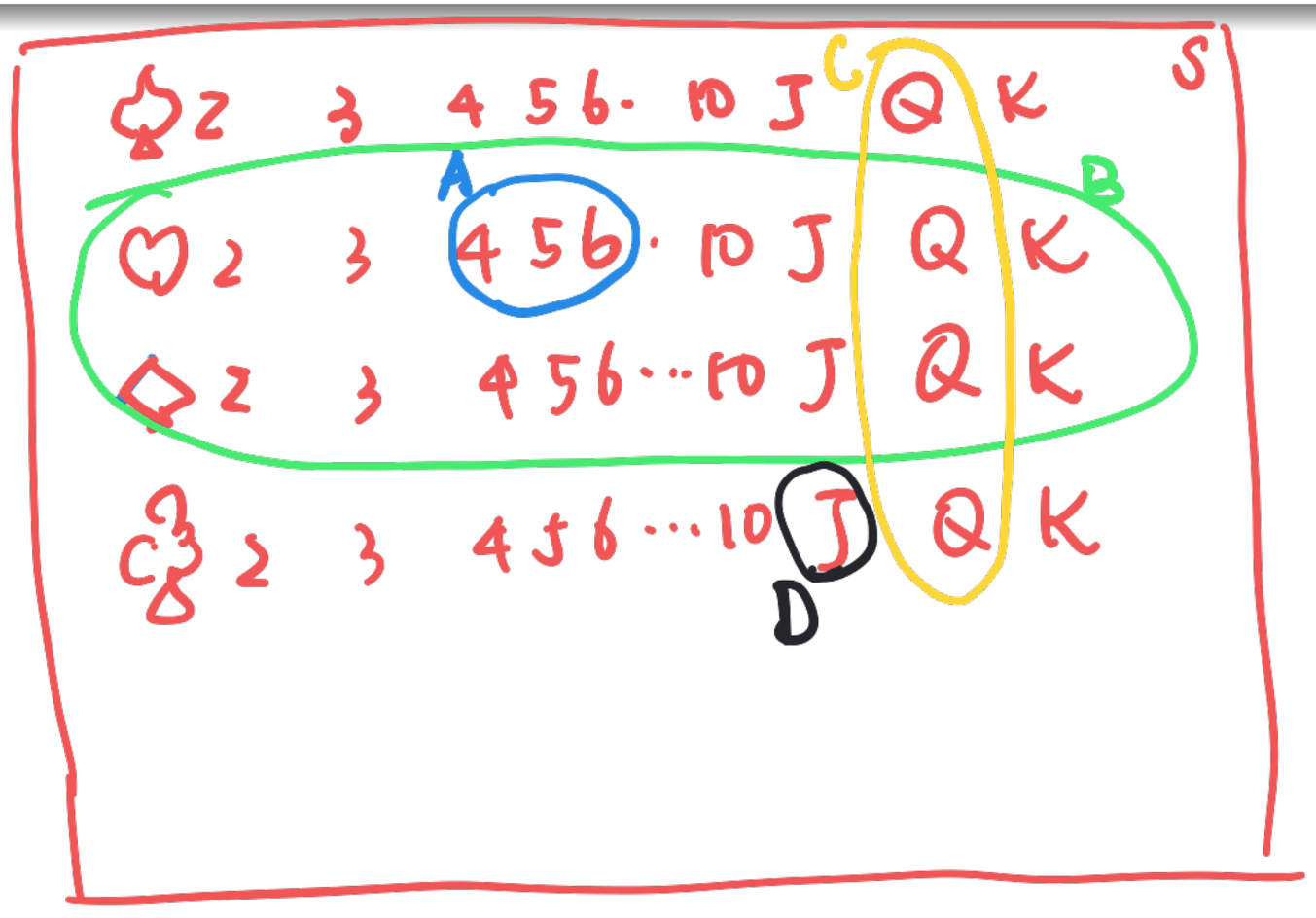
Example 1

Observe the top card drawn on a 52-card deck. Define the events:

A : 4, 5, or 6 of ♡ $\qquad$ B : any red card

C : any queen $\qquad$ D : jack of ♣

Drew a Venn diagram that relates these events to one another.



Thank You

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