

MATH 451/551

Chapter 1. Introduction

1.3.1 Sets

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Set

A **set** is a collection of objects (elements), usually denoted by capital letters such as A, B, \dots .

set builder.

$$\begin{aligned}
 A &= \{1, 2, \dots, 1000\} \\
 B &= \{x \mid x \text{ is a positive integer less than } 101\} = \{1, 2, \dots, 100\} \\
 C &= \{\text{Bulls, Trailblazers}\} \\
 D &= \{(x, y) \mid 0 < x < 1, 0 < y < 2\}
 \end{aligned}$$

Elements

If an object belongs to a set, it is said to be an **element** of the set.

The notation \in is used to denote membership in a set.

\in is a member of
 is an element of
 belongs to

Using the sets defined in the previous example,

$$\begin{aligned}
 17 \in A &\quad 99 \in B & \left(\frac{2}{3}, 1\right) \in D & \text{Cubs} \notin C \\
 \text{17} \in B &\quad 99 \in A
 \end{aligned}$$

Set Operations



Subset

If every element of the set A is also an element of the set B , then A is a **subset** of B . The notation \subset is used to denote the subset relationship.

$$A \subset B \text{ iff } \forall x \in A \Rightarrow x \in B$$

$$\begin{aligned} A &= \{1, 2, \dots, 100\} \\ B &= \{1, 2, \dots, 100\} \end{aligned}$$

The natural numbers N , also known as the positive integers, are a subset of the integers Z , which are a subset of the rational numbers Q , which are a subset of the real numbers R , which are a subset of the complex numbers C . These relationships are compactly stated as

$$N \subset Z \subset Q \subset R \subset C$$

$$N \subset Z \subset Q \subset R \subset C$$

Venn Diagram



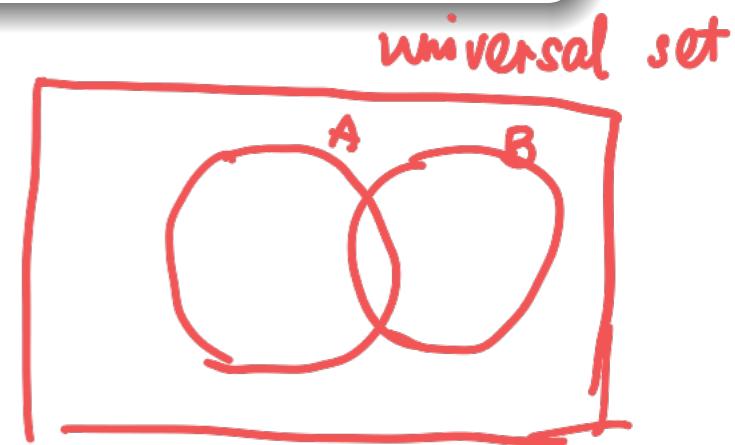
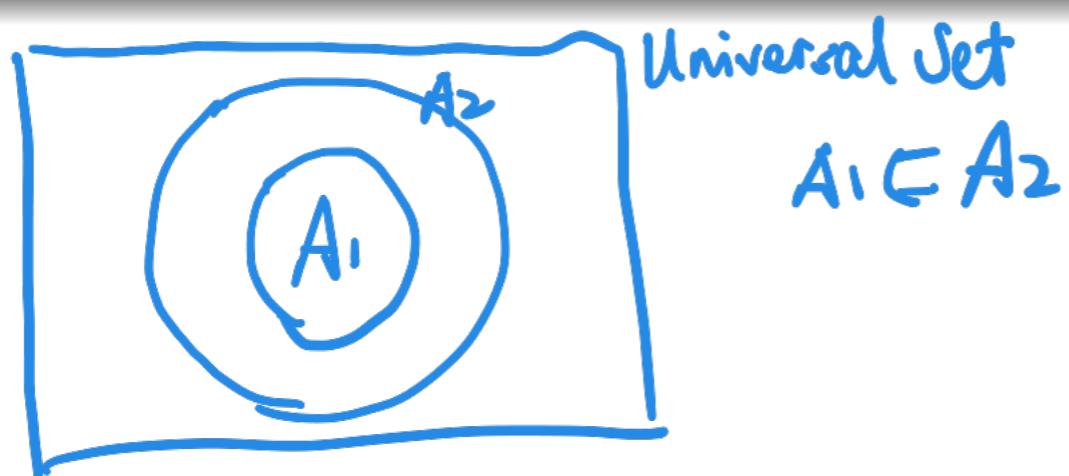
Multiplication Rule

A **Venn Diagram** is a useful tool in set theory and in probability for sorting out the relationships between sets.

- ▶ The external rectangle that is drawn outside of the two sets A and B is called the **universal set**, and it contains all possible elements under consideration.

When there are several subsets involved in a particular application, we often use subscripts, rather than individual letters to denote the sets. Use Venn Diagram to present the relationship between

$$A_1 = \{x | 0 < x < 1\} \text{ and } A_2 = \{x | 0 < x < 5\}.$$



Set Operations (Cont.)



Equality

$A = B$ iff $A \subset B$ and $B \subset A$.

$$\begin{aligned} A \subset B : \forall x \in A \Rightarrow x \in B \\ B \subset A : \forall x \in B \Rightarrow x \in A \end{aligned} \quad \left. \begin{array}{l} \{ \\ \} \end{array} \right\} \Rightarrow A = B$$

Null Set

A set containing no elements is called the **null set** (also known as the **empty set**. The notation \emptyset is used to denote the null set.) \emptyset

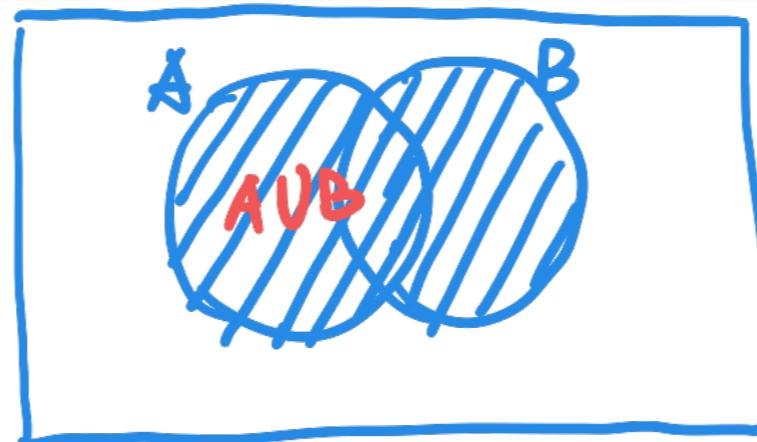
Set Operations (Cont.)



Union

The union of A and B , $\underline{A \cup B}$, is the set of all elements (points) that belongs to either A or B or *both*.

$$\underline{A \cup B} = \{x | x \in A \text{ or } x \in B\}.$$



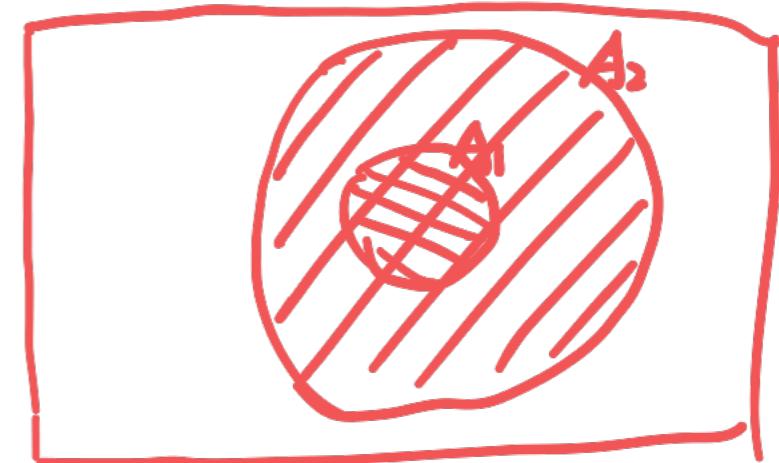
$A_1 \cup A_2 \cup \dots = \cup_{i=1}^{\infty} A_i = \{x | x \in A_1 \text{ or } x \in A_2 \text{ or } \dots\}$, which applies to a finite or infinite number of sets.

Example 1



For the sets $A_1 = \{x | 0 < x < 1\}$ and $A_2 = \{x | 0 < x < 5\}$, find the union of A_1 and A_2 .

$$A_1 \cup A_2 = A_2 = \{x | 0 < x < 5\}$$



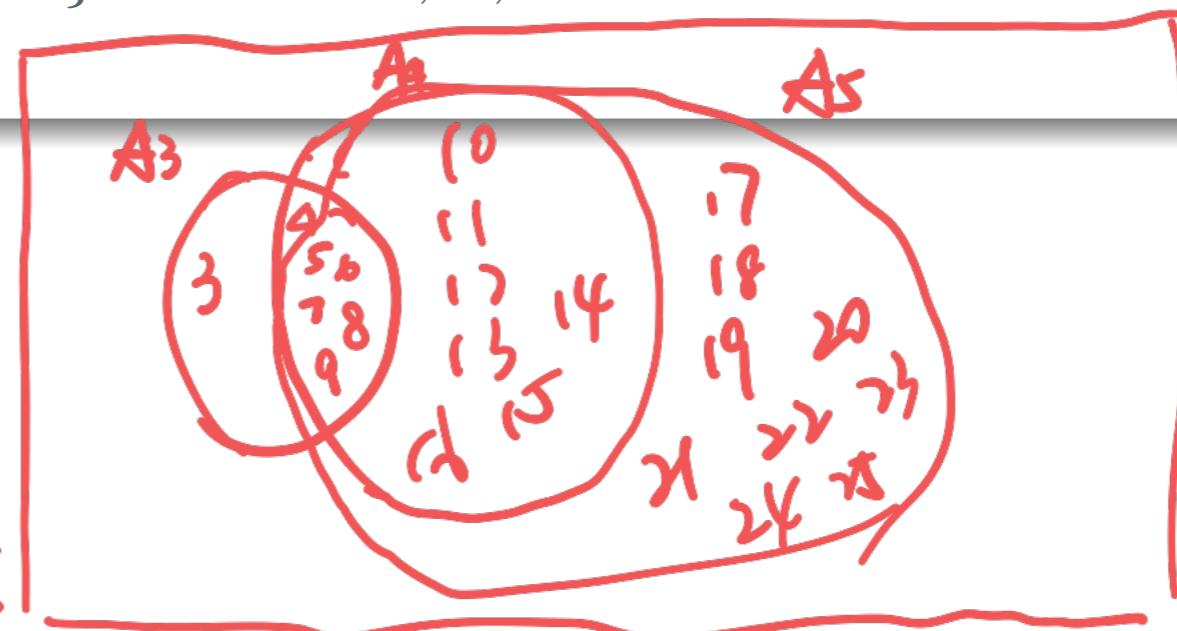
Let $A_k = \{k, k+1, k+2, \dots, k^2\}$ for $k = 1, 2, \dots$. Find the union of A_3 , A_4 , and A_5 .

$$A_3 = \{3, 4, 5, \dots, 9\}$$

$$A_4 = \{4, 5, 6, \dots, 16\}$$

$$A_5 = \{5, 6, 7, \dots, 25\}$$

$$A_3 \cup A_4 \cup A_5 = \{3, 4, 5, \dots, 25\}$$



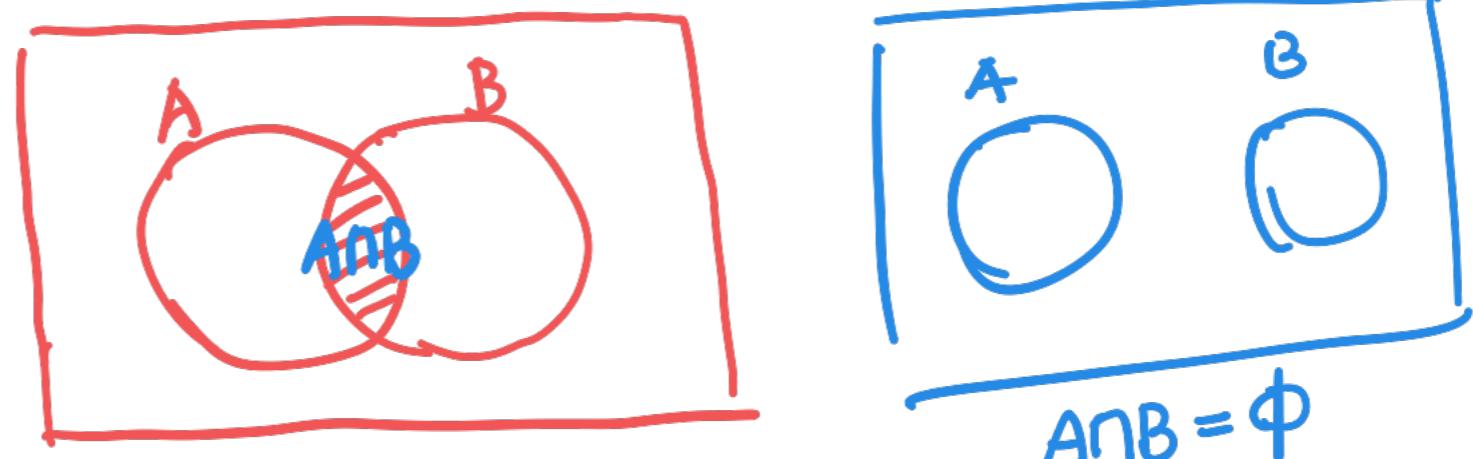
Set Operations (Cont.)



Intersection

The intersection of A and B , $\underline{A \cap B}$, is the set of all elements (points) that belongs to both A and B .

$$\underline{A \cap B = \{x | x \in A \text{ and } x \in B\}}.$$



$\underline{A_1 \cap A_2 \cap \dots} = \underline{\cap_{i=1}^{\infty} A_i} = \{x | x \in A_1, x \in A_2, \dots\}$, which applies to a finite or infinite number of sets.

Example 2



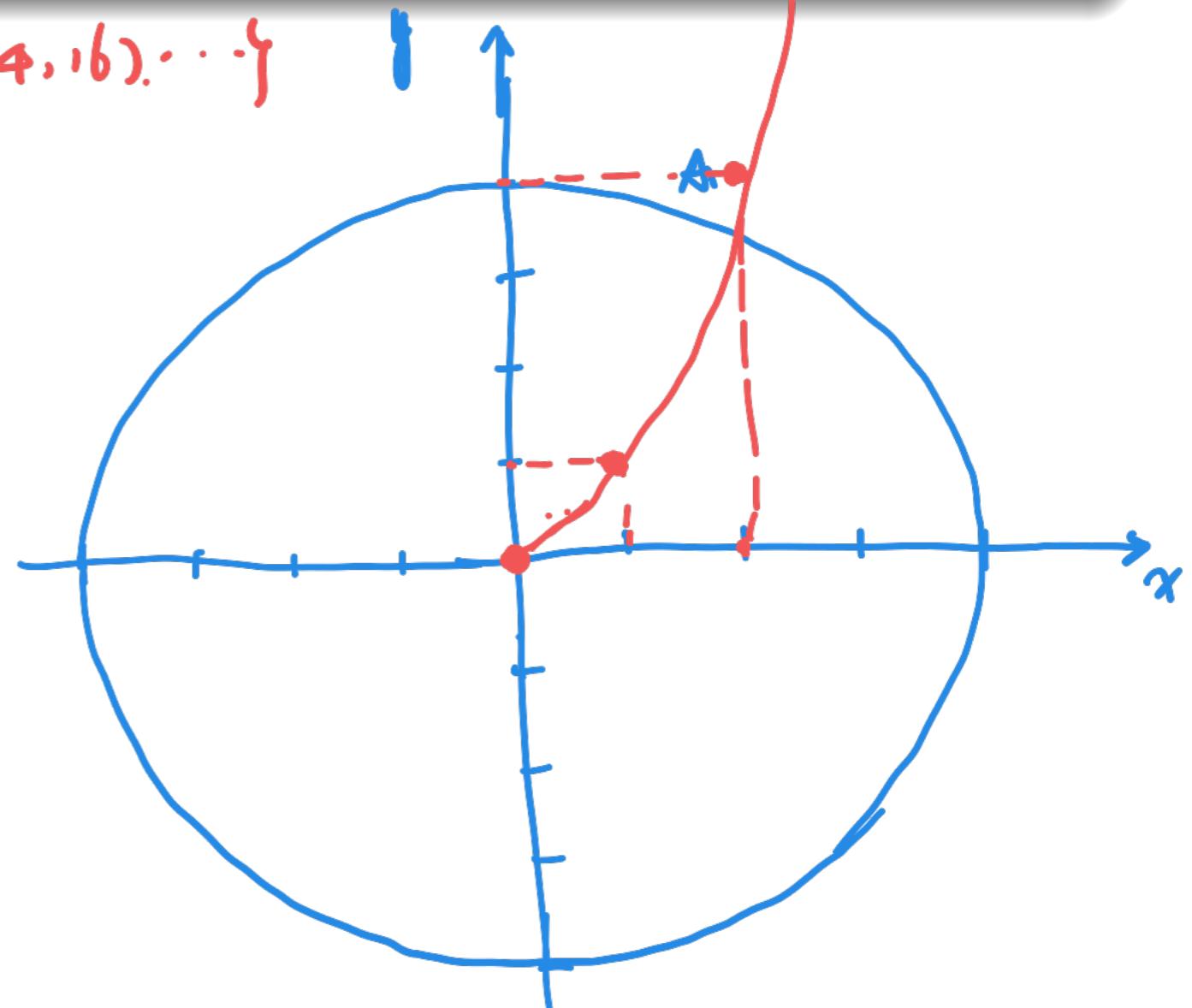
$$\{(3,9), (4,16), \dots\}$$
$$\{(0,0), (1,1), (2,4)\}$$

For the sets $A_1 = \{(x, y) | x^2 + y^2 \leq 16\}$ and $A_2 = \{(x, y) | 0 \leq x \leq 4, 0 \leq y \leq 4\}$, find the intersection of A_1 and A_2 .

$$A_2 = \{(0,0), (1,1), (2,4), (3,9), (4,16), \dots\}$$

$$= \{(k, k^2)\}$$

$$A_1 \cap A_2 = \{(0,0), (1,1)\}$$

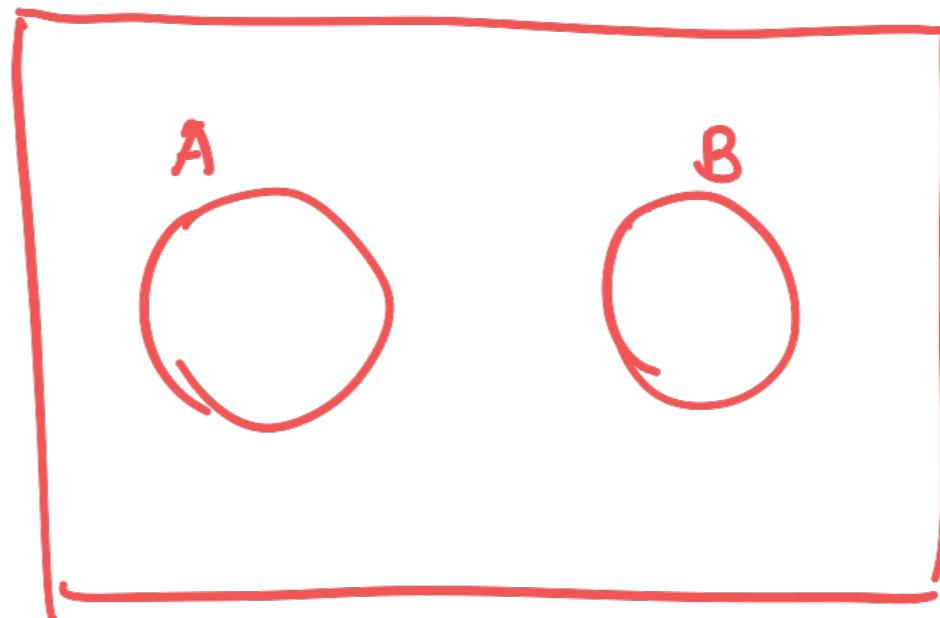


Disjoint (Mutually Exclusive)



Disjoint (Mutually Exclusive)

If $A \cap B = \emptyset$, then A and B are disjoint or mutually exclusive.

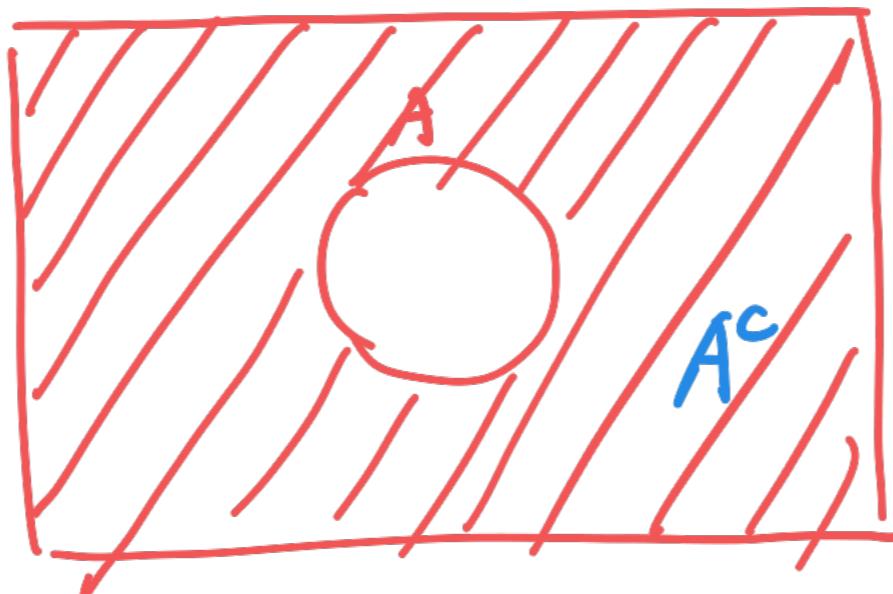




Complement

The **compliment** of A , denoted \bar{A} or A^c , is the set of all elements (points) not in A .

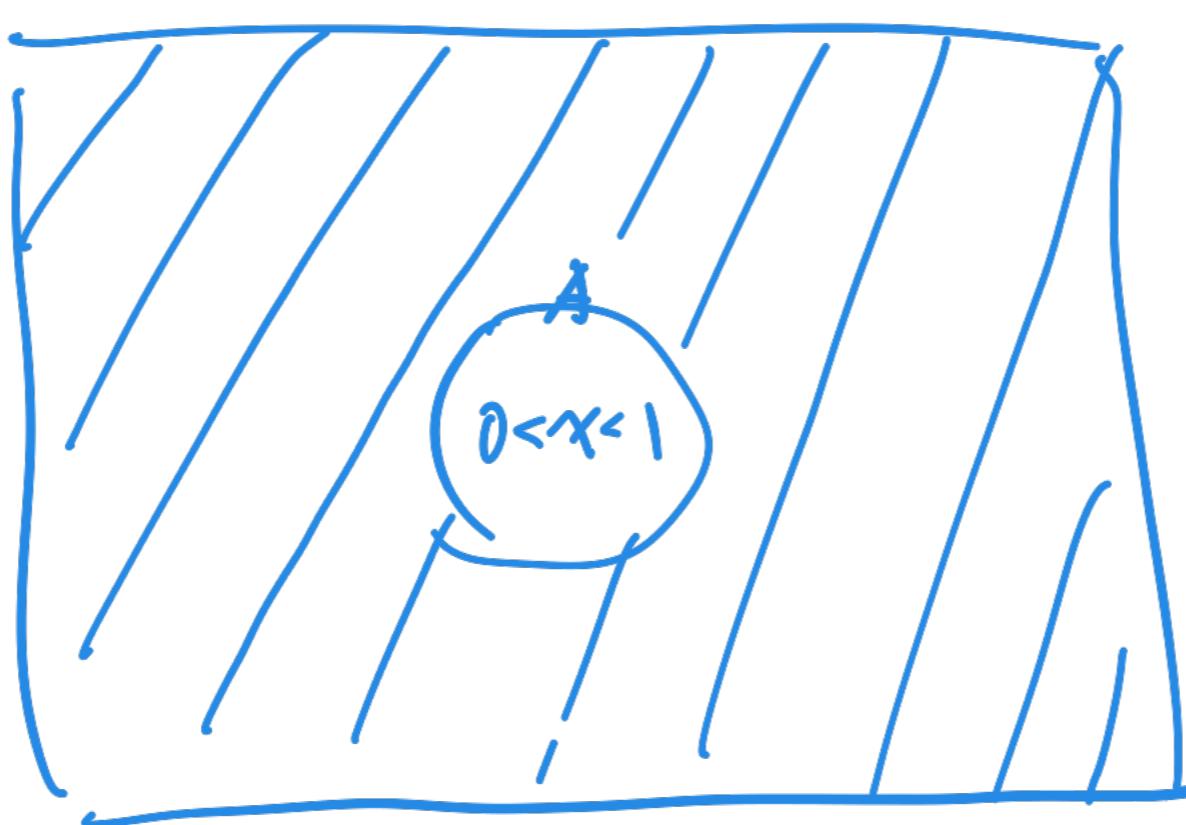
$$\underline{A^c = \{x \mid x \notin A\}}.$$



Example 3



Let the set A be the set of all real numbers on the open interval $(0, 1)$, that is $A = \{x | 0 < x < 1\}$. Find A^c .



$$A^c = \{x | x \leq 0 \text{ OR } x \geq 1\}$$

Example 4



2, ..., 10, J, Q, K, A

2, ..., 10, J, Q, K, A

2, ..., 10, J, Q, K, A



2, ..., 10

J, Q, K, A



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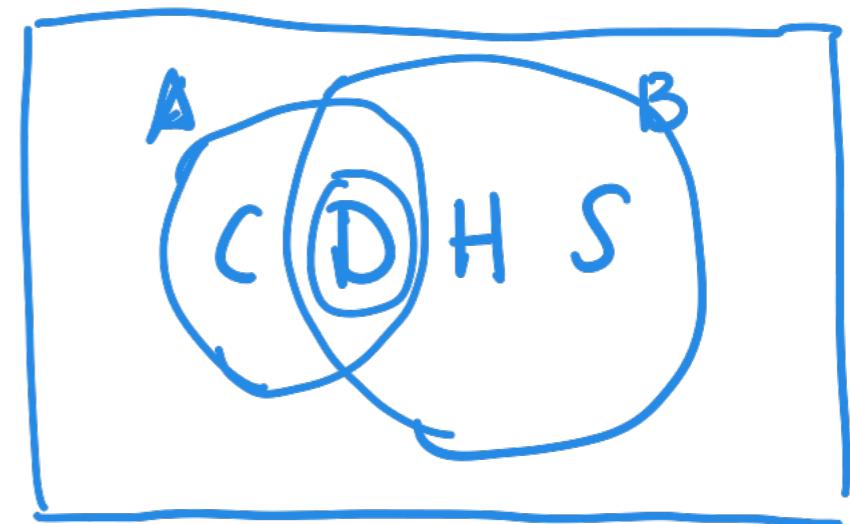
Consider an experiment of selecting a card at random from a standard deck and noting its suit: clubs (C), diamonds (D), hearts (H), or spades (S). All the possible outcomes are $S = \{C, D, H, S\}$. Let $A = \{C, D\}$ and $B = \{D, H, S\}$, find the following sets:

1. $A \cup B = \{C, D, H, S\}$

2. $A \cap B = \{D\}$

3. $A^c = \{H, S\}$

4. $(A \cup B)^c = \emptyset$



Thank You



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THANK YOU!

